REAL-TIME ESTIMATION AND RANDOM SIGNAL DETECTION

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by

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Submitted in partial fulfillment
for the degree of

DOCTOR OF PHILOSOPHY

IN

ELECTRICAL ENGINEERING

from the

UNITED STATES NAVAL POSTGRADUATE SCHOOL

May 1966

ABSTRACT

This work investigates some problems arising in application of (Kalman) linear filter theory to real problems, where practical estimates must replace exact theoretical quantities in problem formulation. The principle objective is application of linear filter theory to random signal detection/classification. However, an example of classical estimation, error estimation in shipboard inertial navigation systems, is offered to illustrate general points discussed. A unified treatment of models for random time series is presented, including a comparative review of models which have been proposed and procedures for obtaining model coefficients. Correlation detection of deterministic signals is discussed and the resulting principles extended to the case of random signal detection. Application of linear filter theory to the problem is indicated. Finally, an experimental study in random signal detection/classification is included. Experimental signals used are hydrophone recordings of sea noise and sea noise plus diesel submarine. Consistency of successful results obtained suggests practical utility of method in certain random signal detection/classification problems.

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LIST OF ILLUSTRATIONS

LIST OF ABBREVIATIONS

$x(t)$	State vector of continuously described linear dynamic system
$y(t)$	Observation vector of system
$z(t)$	Vector of noisy observations, equals $y(t) + v(t)$
$w(t)$	White noise excitation vector
$v(t)$	Vector of white noise contaminating observations
$F(t)$	Matrix describing the homogeneous system
$G(t)$	Input matrix of linear dynamic system
$H(t)$	Output matrix of system
Q	Covariance matrix of excitation noise vector $w(t)$
q	Variance of scalar valued excitation noise
R	Covariance of vector valued observation noise
r	Variance of scalar valued observation noise
z^{-1}	Unit delay
$x(k)$	State vector of discretely described linear dynamic system
$y(k)$	Observation vector of discrete system
$z(k)$	Vector of noisy discrete observations
$w(k)$	Vector of white noise excitation
$v(k)$	Vector of white noise contaminating observations
$\Phi(k)$	State transition matrix of discretely described linear dynamic system
$\Gamma(k)$	Input matrix for discrete system
$H(k)$	Output matrix for discrete system
S	Covariance of system state vector
$x(k/j)$	Optimal estimate of state vector $x(k)$ given data to time j
$P(k/j)$	Covariance of error of optimal state estimate at time k given data to time j
$B(k)$	Kalman filter gain matrix
n	Order of linear dynamic system

N	Number of signal samples to be processed
$R(\tau)$	Autocorrelation function
I	Identity Matrix

ACKNOWLEDGEMENT

The author wishes to express sincere appreciation to Professor Harold A. Titus for his guidance and encouragement throughout the period of this research.

CHAPTER I

INTRODUCTION

It is frequently an integral requirement in engineering information processing systems that measurement data be employed in subsequent decision making or action. Invariably, the processing problem lies in a stochastic environment because of inaccuracies in the measurement process or random perturbations of the observed process during or between measurements. Some examples are determination of satellite orbit parameters from noisy radar or radio-location data, target track determination in the radar fire control problem, filtering of signals received in the presence of noise, and state estimation in linear dynamic systems from noisy transducer information.

What might be termed "classical" estimation theory in engineering has been based upon the pioneering work of Wiener published in 1942. This theory rests upon three main assumptions: (i) linear operations on received data, (ii) a quadratic loss criterion, and (iii) stationary statistics describing the signal and noise processes. In applications, the objective is to obtain a specification of the electrical filter (the Wiener filter) giving optimum estimation, or isolation, of the signal from corrupting additive noise. Posed in terms of integral equations, the problem results in the Wiener-Hopf integral equation, which must be solved to obtain the impulse response of the optimum filter. Broad application of the theory has suffered from the difficulty of obtaining solutions for the Wiener-Hopf equation.

In 1960, Kalman posed the estimation problem in terms of differential equations, using the concept of "state" from linear system theory. Characterization of the problem in terms of differential equations offers several advantages. Chief among them are suitability of the new approach for machine computation, relaxation of the stationarity assumption, and the intimate relationship between the optimum filter and a linear model describing the signal

process. The work here is motivated by the contributions of Kalman and considers application of his techniques to problems in statistical communication theory.

In the title, the term "real-time" means that computational effort for estimation procedures to be employed does not increase with increased numbers of observations. Further, following the practice of recent years, the collective term "estimation" is considered to include the traditional problems of smoothing, filtering, and prediction. Since digital computation is to be employed in examples, and since the required mathematics is less obscure, the present work will deal almost exclusively with estimation of discrete signals. For engineering purposes, little generality is lost. The description of continuous filters can in general be obtained by considering the limiting case of the corresponding discrete filter as the sampling interval is allowed to approach zero.

In Chapter II, basic assumptions underlying optimum estimation theory developed by Kalman are reviewed, and some practical considerations are discussed. As an example, the problem of estimating latitude error in shipboard inertial navigation systems is outlined, and application of the Kalman filter to the problem is described.

Chapter III considers the important problem of finite parameter models for random time series, a subject not heretofore treated in a unified manner. First, the Spectral Distribution Function, introduced for the discrete time case by Wold in 1938, is discussed and its three constituent distribution functions noted. Then models which have been proposed for time series are considered with regard to their capacities for representing the various constituent functions. The combined autoregressive-moving average model is shown to be equivalent to the linear filter, and the relationship between their respective parameters is indicated. A discussion of parameter estimation is then included. Present difficulties in solving for moving average model parameters is noted. It is shown that the approach of

Ho and Lee [16] to the identification problem amounts also to a recursive means for estimating autoregressive model coefficients. Finally, the recursive approach is employed in an example to obtain autoregressive model coefficients for a sample signal. The signal consists of a 50 cycle sine wave combined additively with noise from a laboratory random noise generator. A plot of z-plane poles of the model is given. Also included are comparative graphs of the auto-correlation functions and power spectral densities derived from the signal and model respectively.

The detection of random signals in noise is considered in Chapter IV. As it happens, optimum detection of Gaussian random signals constitutes a simple (at least conceptually) extension to the idea of correlation detection of deterministic signals. In the case of Gaussian random signals, the optimum receiver forms a cross correlation of the received signal and a least squares estimate of the Gaussian signal given the received signal. The extension is shown following a brief development of correlation detection of deterministic signals. Difficulties involved in directly evaluating the Likelihood Function for each alternative classification are noted. Then employing the approach of Schweppe [31], it is shown that the Likelihood Function may be evaluated recursively. For random signals which are single time series, no matrix inversion is required. Quantities required in the recursion relations may be obtained from Kalman filter estimation of the Gaussian signal given the received signal. On-line computations may be greatly reduced by storage of short lists of required quantities which do not depend upon the received signal. Using stored lists, on-line multiply-add combinations per received signal sample are reduced to approximately $2n + 2$ for each classification alternative, where n is the order of signal models used.

An experimental problem in random signal detection investigated in Chapter V represents the first testing against actual signals of the methods developed or discussed in this dissertation. The experimental

signals used are hydrophone recordings of (a) a diesel submarine plus sea noise and (b) sea noise alone. After obtaining autoregressive models for the signal-plus-noise and noise processes, recursive Likelihood Function evaluation discussed in Chapter IV is employed to test performance of optimum linear classification on actual signals.

Consistent correct classification is obtained within less than 100 samples of signal record (representing less than 60 millisecs of signal record). Furthermore, classification efficiency is maintained over rather wide deviations between actual and assumed received signal parameters. Hence initial results suggest practical applicability of the method to certain real-time random signal detection/classification problems.

CHAPTER II

LINEAR LEAST SQUARES ESTIMATION - THE KALMAN FILTER

Interest in time-domain estimation techniques and applications has been stimulated in recent years by the contributions of R. E. Kalman [19]. Subsequent investigators have related Kalman's derivation and results to more familiar estimation techniques, thereby enhancing general understanding and utility of the new approach. Perhaps the simplest derivation of Kalman's results lies in the Bayesian approach employed by Peschon and Weaver [26], [35]. Another step toward better perspective was the demonstration by Lee [21] and Fagin [10] of the equivalence between Kalman's results and generalized recursive least squares estimation. A unified and general treatment of modern estimation theory by Deutsch [9] has recently been published.

The purpose of this chapter is to review the basic assumptions underlying Wiener-Kalman filter theory and implications of the assumptions. Further, practical considerations arising in applications are discussed. In this regard, a comparative discussion of the Kalman filter and generalized, recursive estimation of a Gaussian population mean is introduced to explicitly show the role of a priori information. Finally, a practical problem is considered to illustrate an application of the methods treated here.

I. BASIC ASSUMPTIONS

The two basic assumptions underlying optimal filter theory are (1) linear operations on the data are to be performed, and (2) the criterion for goodness is mean squared error. Under these assumptions, it is well known that only second order statistical moments are employed in the estimation scheme [8]. An immediate result is that for non-Gaussian signal processes, the optimal linear estimate obtained is the same as would have been obtained from the corresponding Gaussian signal process (i.e. possessing the same first

two moments). Moreover, the optimal linear least squares estimate of a Gaussian process is identical with the Maximum Likelihood estimate. The conclusion follows that linear least squares estimation may be improved upon only for non-Gaussian processes, and then only by an estimation procedure which takes into account third or higher statistical moments of the process. Conversely, in discussions restricted to linear least squares estimation, assumptions of Gaussian process statistics can be made without effect on the estimation outcome.

II. THE SIGNAL PROCESS MODEL

It is commonly assumed in engineering literature on stochastic estimation that the scalar or vector-valued random time series to be processed can be regarded as the output of a linear dynamic system excited by white noise. Implications of this model will be discussed in greater detail in Chapter III. It is also assumed that observations of the system output are obscured by additive Gaussian white noise. Block diagrams of continuous and discrete descriptions of the model are given in Figs. 2-1 and 2-2 respectively.

The general continuous linear dynamic system model may be described by the vector differential equations

$$\dot{x}(t) = F(t)x(t) + G(t)w(t)$$

$$y(t) = H(t)x(t)$$

$$z(t) = y(t) + v(t)$$

where

$x(t)$ is the $n \times 1$ vector of system states.

$F(t)$ is an $n \times n$ matrix describing the homogeneous system.

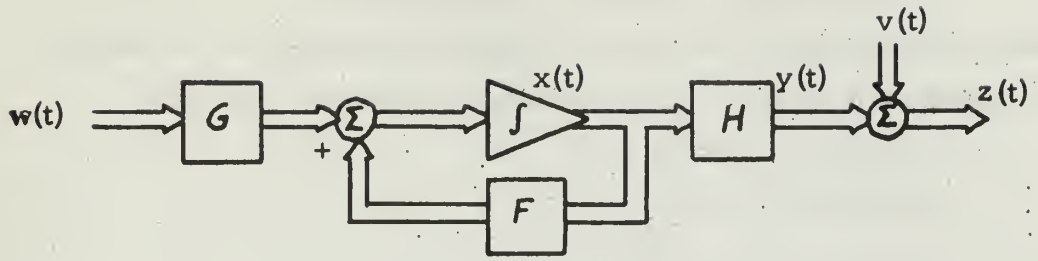
$w(t)$ is a $p \times 1$ vector of Gaussian white noise

$G(t)$ is an $n \times p$ matrix which distributes excitation noise across the states.

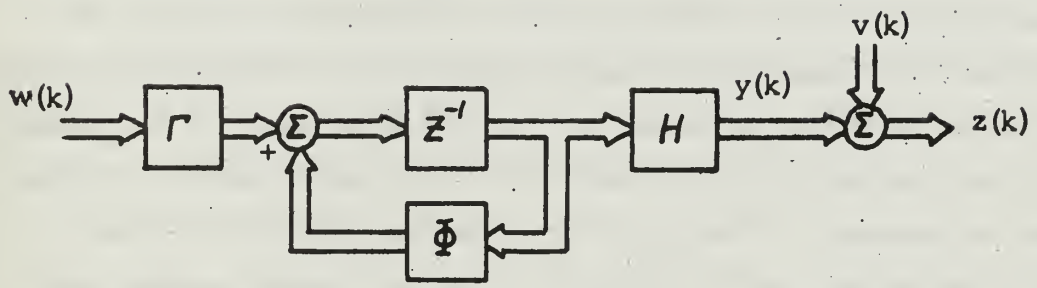
$y(t)$ is an $m \times 1$ vector of system output.

$H(t)$ is an $m \times n$ observation matrix.

$v(t)$ is an $m \times 1$ vector of white Gaussian measurement noise



Block Diagram of General Continuous
Linear Dynamic System
Fig. 2-1



Block Diagram of General Discretely
Described Linear Dynamic System
Fig. 2-2

$z(t)$ is an $m \times 1$ vector of observations.

Excitation and measurement noise sources are assumed to be independent. Characteristics of the noise vectors $w(t)$ and $v(t)$ are:

$$E[w(t)] = E[v(t)] = 0$$

$$E[w(t)w(\tau)^T] = \begin{cases} Q & t = \tau \\ O & t \neq \tau \end{cases}$$

$$E[v(t)v(\tau)^T] = \begin{cases} R & t = \tau \\ O & t \neq \tau \end{cases}$$

For the corresponding discretely described linear dynamic system, the describing vector difference equations are

$$x(k) = \Phi(k)x(k-1) + \Gamma(k)w(k-1)$$

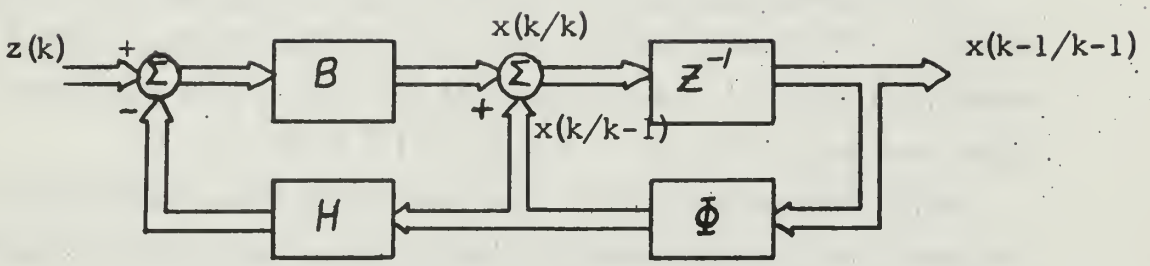
$$y(k) = H(k)x(k)$$

$$z(k) = y(k) + v(k)$$

Corresponding to the continuous case, the noise vectors $w(k)$ and $y(k)$ are assumed to be samples of a vector-valued Gaussian noise process, and are assumed to be independent from sample to sample. For $w(k)$, this assumption of uncorrelated samples represents no loss of generality, since the excitation noise is fictitious anyway. On the other hand, measurement noise encountered in certain applications may not be totally uncorrelated from sample to sample. In such cases, the correlated portion may be accounted for by augmenting the system order as necessary. The form of the resulting augmented system is simply arrived at. Actual numbers to describe the measurement error correlation may be obtained by considering the points discussed in Chapter III.

III. THE OPTIMAL FILTER

References cited above include derivations of the Kalman filter employing variously geometric (orthogonal projections in Hilbert space), Bayesian, and Maximum Likelihood approaches. Additionally, equivalence of the optimal filter and recursive linear least squares prediction has been indicated. It will therefore suffice for the work to follow to include here only the equations describing the optimal filter and the corresponding block diagram (Fig. 2-3). The



Block Diagram of Kalman Filter

Fig. 2-3

equations given are for the discrete formulation.

$$B(k) = P(k/k-1)H^T(HP(k/k-1)H^T + R)^{-1}$$

$$x(k/k) = \Phi x(k-1/k-1) + B(k)[z(k) - H \Phi x(k-1/k-1)]$$

$$P(k/k) = [I - K(k)H]\,P(k/k-1)$$

$$P(k+1/k) = \Phi P(k/k)\,\Phi^T + \Gamma Q \Gamma^T$$

In the equations given, P is an n x n covariance matrix of the estimate. The double index is used to mean that P(k/k-1) is the variance of x(k/k-1), the optimal state estimate at time increment k given data only up to time k-1 ($x(k/k-1) = \Phi x(k-1/k-1)$).

While the equations above describe optimal linear estimation of the signal process model states, what frequently is required is the optimal estimate of some linear combination of the states, for example $y = Hx$. In order that the equations above be useful in such a case, it must be so that the optimal estimate of a linear combination of states is the same linear combination of the optimal estimate of the states. A proof of this point is included in Appendix I.

IV. A PRIORI INFORMATION

As is readily noted, the difference equations above which describe behavior of the Kalman filter or recursive least squares estimation require a starting point. What is needed is an initial estimate of the states x(1/0), and an initial covariance matrix P(1/0), indicating the uncertainty in the initial estimate. These quantities represent a priori knowledge about the process to be estimated since they are required to be specified before any observations are made. An intuitive feel for the role of x(1/0) and P(1/0) may be gained by comparing behavior of the Kalman filter and generalized recursive estimation of a Gaussian population mean, outlined in Appendix II. It is shown there that a priori information appears to the filter just as additional equivalent observations of the process to be estimated.

It is difficult in many problems to arrive at meaningful values for

the a priori quantities required. If the signal process model is stable and the excitation noise covariance Q is known, a priori knowledge may be obtained directly from knowledge of the model. Motion of states of the model is a Gaussian random process, a linear function of the Gaussian excitation process. Since the excitation has zero mean, the expected value of states of the model given no observations is also zero. Thus $x(1/0)$ equals the zero vector. Further, the covariance describing uncertainty in state values is the solution matrix S of the difference equation

$$S = \Phi S \Phi^T + \Gamma Q \Gamma^T$$

At any stage, the matrix $P(k/k-1)$ is defined as the error covariance matrix at time k given observations to time $k-1$. Hence

$$\begin{aligned} P(1/0) &= E[(x(1) - x(1/0))(x(1) - x(1/0))^T] \\ &= E[x(1)x(1)^T] \\ &= S \end{aligned}$$

Another suggested approach when a priori requirements present difficulty is to specify some reasonable value of $x(1/0)$ and let $P(1/0) = cI$, where c is a large number. It is motivated by the fact that weighting of measurement data is in inverse proportion to uncertainty in the data relative to uncertainty of the current best estimate before data receipt. Thus effects of the initial assumptions may be washed out by the incorporation of new data [21], [10].

In the example below, both the above methods are tried and comparative results noted. Further use of the two methods is made in the experimental study of Chapter V. There the latter method is used in the recursive model determination program, and the former is used in the program for computation of Likelihood Functions.

V. AN EXAMPLE - ESTIMATION OF SYSTEM ERROR IN SHIPBOARD INERTIAL NAVIGATION SYSTEMS

An example to which the techniques described above may be very naturally applied is the problem of error estimation in shipboard inertial

navigation systems (SINS). Gyroscopes used in the system to provide a stable inertial platform are subject to certain random drifts. Hence in order to maintain the necessary accuracy of the system over indefinite periods at sea, it is necessary to determine system errors for use in recalibrating or resetting the SINS. Such an error determination necessarily depends upon information from external sources; in this case upon LORAN position information and position/heading information from star sightings, etc. However, information from these sources is imperfect due to measurement inaccuracies. Additionally, it is desired to have to resort to LORAN or other observations as infrequently as possible, so optimal use of observational data is indicated. These conditions, optimal estimation from noisy observations, comprise justification for considering the problem here.

The problem will be pursued here only far enough to illustrate application of the method. Thus only the problem of estimating latitude error will be discussed in detail. Without further theoretical complication, the treatment may be expanded to encompass remaining aspects of the error estimation problem. Optimal control techniques may then be employed in system reset, a step under current development [5].

1. Background

As stated above, SINS employs gyroscopes to stabilize a gimbal-supported inertial platform. Remarks to follow will relate to the four-gimbal system[1], the innermost two gimbals of which are termed the "latitude gimbal" and "heading gimbal." Upon the innermost gimbal, the latitude gimbal, are mounted three stabilizing single-degree-of-freedom gyros with mutually perpendicular input axes.

[1]The present discussion will be only detailed enough to outline the estimation problem to be considered. More detailed discussion of inertial navigation systems may be found in [24], [32].

This basic structure will be discussed further below.

Two coordinate systems fixed in the gimbals describe operation of the system. They have been termed the "instrumented equatorial coordinate system", fixed in the latitude gimbal with axes E', Y', and P', and the "instrumented local coordinate system", fixed in the heading gimbal with axes x', Y', and z'. These axes correspond to "earth axes" E, Y, P, x, and z shown in Fig. 2-4. The latitude and heading gimbals are pivoted to each other on the Y' axis.

Identification of axes shown in Fig. 2-4 follows:

E is parallel to intersection of local meridian plane with plane of equator, positive outward.

Y is in local horizontal plane and extends eastward.

P is parallel to polar axis of earth, positive toward North pole.

x is in local horizontal plane and extends northward.

z is parallel to local vertical, positive downward.

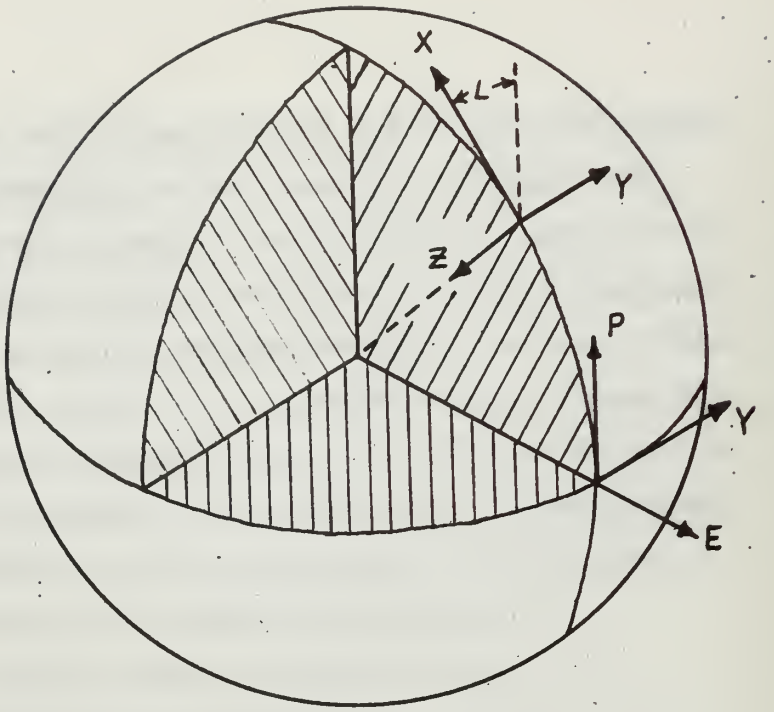
Having introduced the coordinate systems employed in SINS, orientation of the stabilizing gyros may be more completely described. As stated, they are mounted on the innermost, or latitude gimbal, in which is fixed the instrumented equatorial coordinate system. The gyros are termed the equatorial, latitude, and polar gyros, and are mounted with their input axes parallel to the E', Y', and P' axes respectively. Drift rates of the three gyros are denoted, respectively, by ϵ_E, ϵ_L, and ϵ_P.

In order that the instrumented equatorial coordinate system correspond accurately with the earth's equatorial coordinate system, the latitude gimbal is torqued about the P' axis at the rate $\Omega + \lambda$ where

Ω= Earth rate

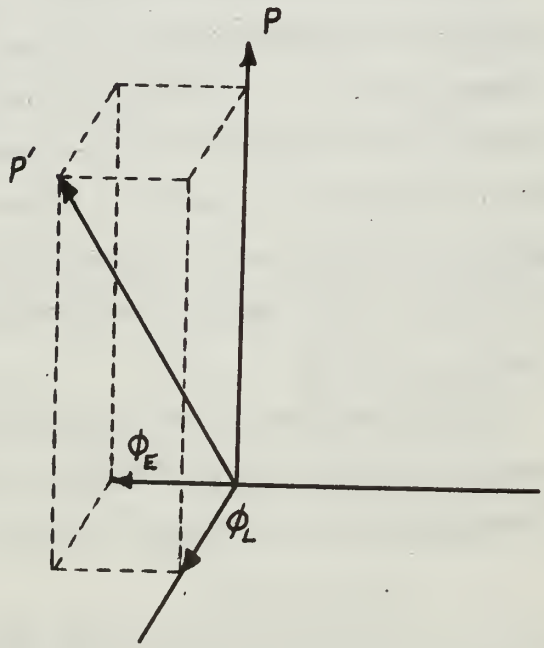
λ = Ship longitude rate obtained via an accelerometer on the inertial platform.

Additionally, the heading gimbal is torqued with respect to the latitude gimbal at a rate (-L') equal to the negative of the SINS indicated



Equatorial and Local Coordinate Systems

Fig. 2-4



Coordinate System Misalignment

Fig. 2-5

latitude rate (also obtained via an accelerometer on the inertial platform). In operation, the system indicated latitude is given by the angle between the latitude and heading gyros, and indicated longitude is given by the angle between the latitude gimbal and a reference.

This completes a brief description of system operation.

2. Error Generation

For the present discussion, operation is assumed to be in "damped inertial" mode, reducing Schuler oscillation errors to comparatively insignificant dimensions. Nevertheless, it remains necessary to estimate errors resulting from imperfect system resets and the gyro drifts ϵ_E, ϵ_L, and ϵ_P mentioned above.

When the system is misaligned, the instrumented and earth equatorial coordinate systems do not coincide as desired. Misalignment consists of small angular displacements ϕ_L about the Y axis and ϕ_E about the E axis. The situation is depicted in Fig. 2-5, where the vector P' has unit length.

In the absence of gyro drift, the P' axis is motionless relative to inertial space, while the earth coordinate system rotates about P at the earth rate. The projection of P' in the equatorial plane thus describes a circle centered at the origin and repeated every 24 hours. Differential equations describing the circle are

$$\dot{\phi}_L = -\Omega \phi_E$$

$$\dot{\phi}_E = \Omega \phi_L$$

Introducing gyro drift, ϕ_L acquires an additional rate of change equal to ϵ_L, the latitude gyro drift. Similarly, ϕ_E acquires an additional rate of change equal to ϵ_E caused by equatorial gyro drift. The equations become

$$\dot{\phi}_L = -\Omega \phi_E + \epsilon_L$$

$$\dot{\phi}_E = \Omega \phi_L + \epsilon_E$$

If it is assumed that the drift rates ϵ_L and ϵ_E are constant, the

equations may be written

$$\frac{d}{dt}\left(\phi_L + \frac{\epsilon_E}{\Omega}\right) = -\Omega\left(\phi_E - \frac{\epsilon_L}{\Omega}\right)$$

$$\frac{d}{dt}\left(\phi_E - \frac{\epsilon_L}{\Omega}\right) = \Omega\left(\phi_L + \frac{\epsilon_E}{\Omega}\right)$$

The locus is still a circle but is centered off the origin as shown in Fig. 2-6.

3. Error Estimation

Importance of the error locus lies in its relation to physically measurable errors. Under the common assumption of perfect knowledge of the local vertical (i.e. z' and z vertical), it has been shown [32] that

$$\phi_L = \zeta L$$

$$\phi_E = \zeta H \cos L$$

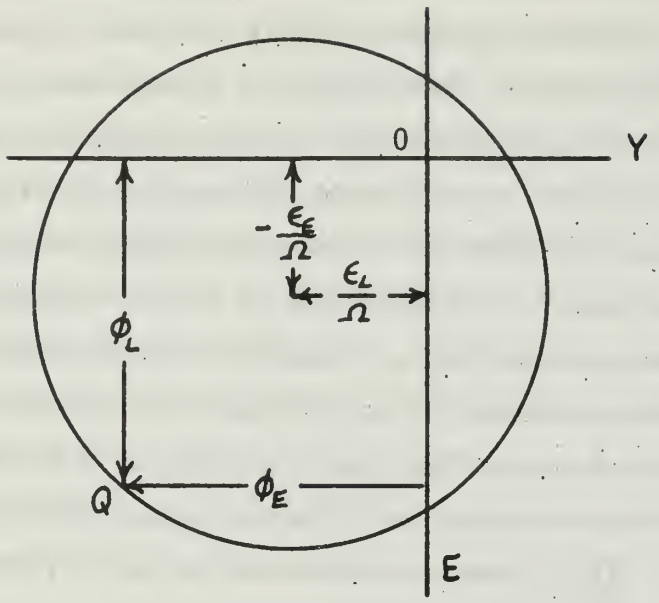
where

ζL = Latitude error

ζH = Heading error

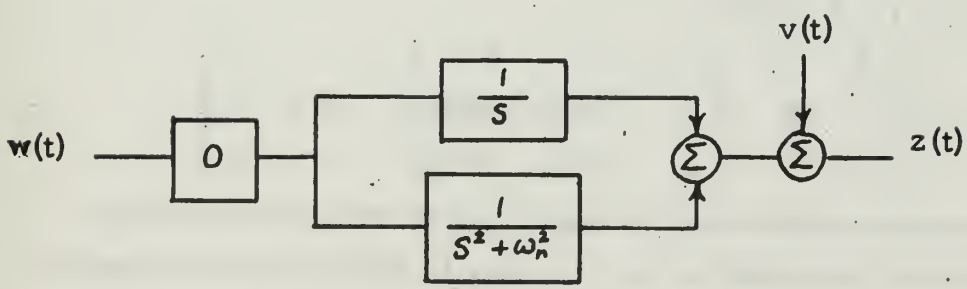
In general, only position information is conveniently and regularly available. Hence only ϕ_L can be directly estimated. By use of three or more latitude error observations, the circle may be determined to within the Y axis displacement $\frac{\epsilon_L}{\Omega}$.

Current operating procedure involves a manual graphical fit of a 24 hour period sine wave through plotted succeeding values of latitude error versus time. The method is patently both time consuming and vulnerable to human error. In the discussion below, this same reduced problem will be treated with a recursive least squares procedure (Kalman filter). While, as stated, no further theoretical complication is involved if treatment is extended to the complete problem, all essential features needed to show the application are present in the reduced problem unobscured by arithmetic.



Error Locus with Constant Gyro Drifts

Fig. 2-6



Linear Model for Latitude Error

Fig. 2-7

4. Recursive Least Squares Error Estimation

In order to employ the Kalman filter, it is necessary to specify a model for the observed process in terms of a linear dynamic system excited by white noise. Furthermore, it is necessary to possess a priori information about the amount of noise contaminating observations and about the mean squared value of the observed process.

Since only estimation of latitude error (which equals Φ_L) is to be considered in the reduced problem, a simple form for the model may be written by inspection. It appears in block diagram form in Fig. 2-7. The error locus of Fig. 2-6 may be considered to be projected upon the E axis. Hence error estimation in the reduced problem amounts to estimation of the unknown amplitude, phase, and offset $\left(-\frac{\epsilon_E}{\Omega}\right)$. The singular model of Fig. 2-7 has a null G matrix, and its behavior is thus determined solely by the unknown initial conditions. In Fig. 2-7, the pure integrator leg preserves the initial offset, and the second order leg represents the 24 hour period sine wave of unknown amplitude and phase. Using this model, a priori quantities used in the filter are $x(1/0) = 0$ and $P(1/0) = 10{,}000I$. The model of Fig. 2-7 was converted to discrete form suitable for digital computation. The resulting matrices describing it are given below (for $T = 1$ hour).

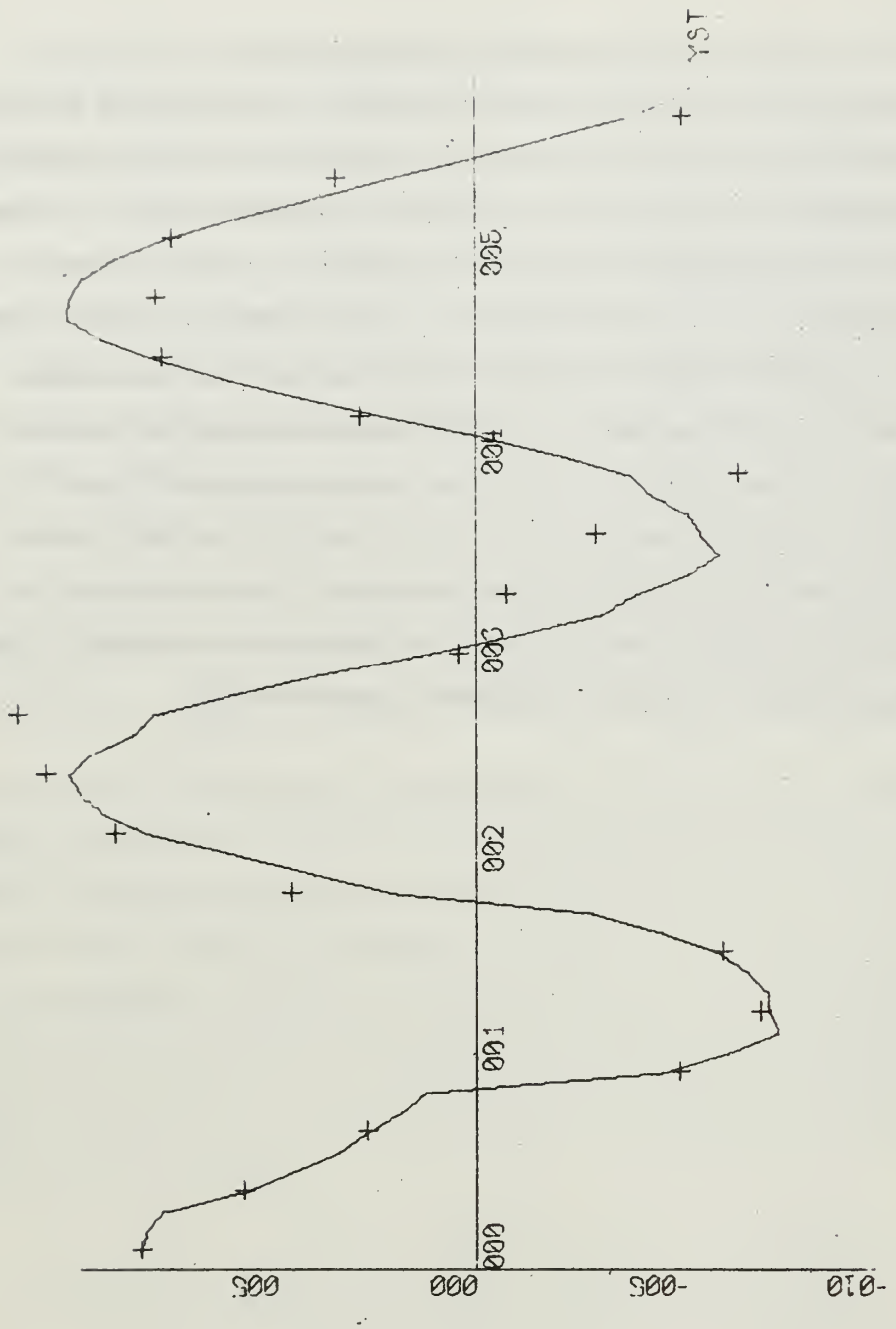
$$\Gamma = \begin{bmatrix} 0 \\ \end{bmatrix} \quad \Phi = \begin{bmatrix} 1 & 0 & 0 \\ 0 & .9659 & .9886 \\ 0 & -.0678 & .9659 \end{bmatrix} \quad H = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

In order to obtain a priori quantities in the alternate manner noted above, the model of Fig. 2-7 may be made absolutely stable by the addition of damping in each leg. The precise 24 hour period of oscillation and essentially constant offset (constant drift rate) suggests that only slight damping be used. Then a value of q, variance of the white excitation noise, is specified such as to obtain steady state peak oscillation of about 5-10 minutes of arc,

a rough mean peak oscillation observed in practice.

In the simulation, latitude measurements were assumed available every three hours, and were all assumed to be of the same quality. Assumed standard deviation of measurement noise was five. It was assumed that an updated latitude error estimate, with its corresponding variance, was desired hourly. Hence updating without data is done in the intervening hours between receipt of measurements.

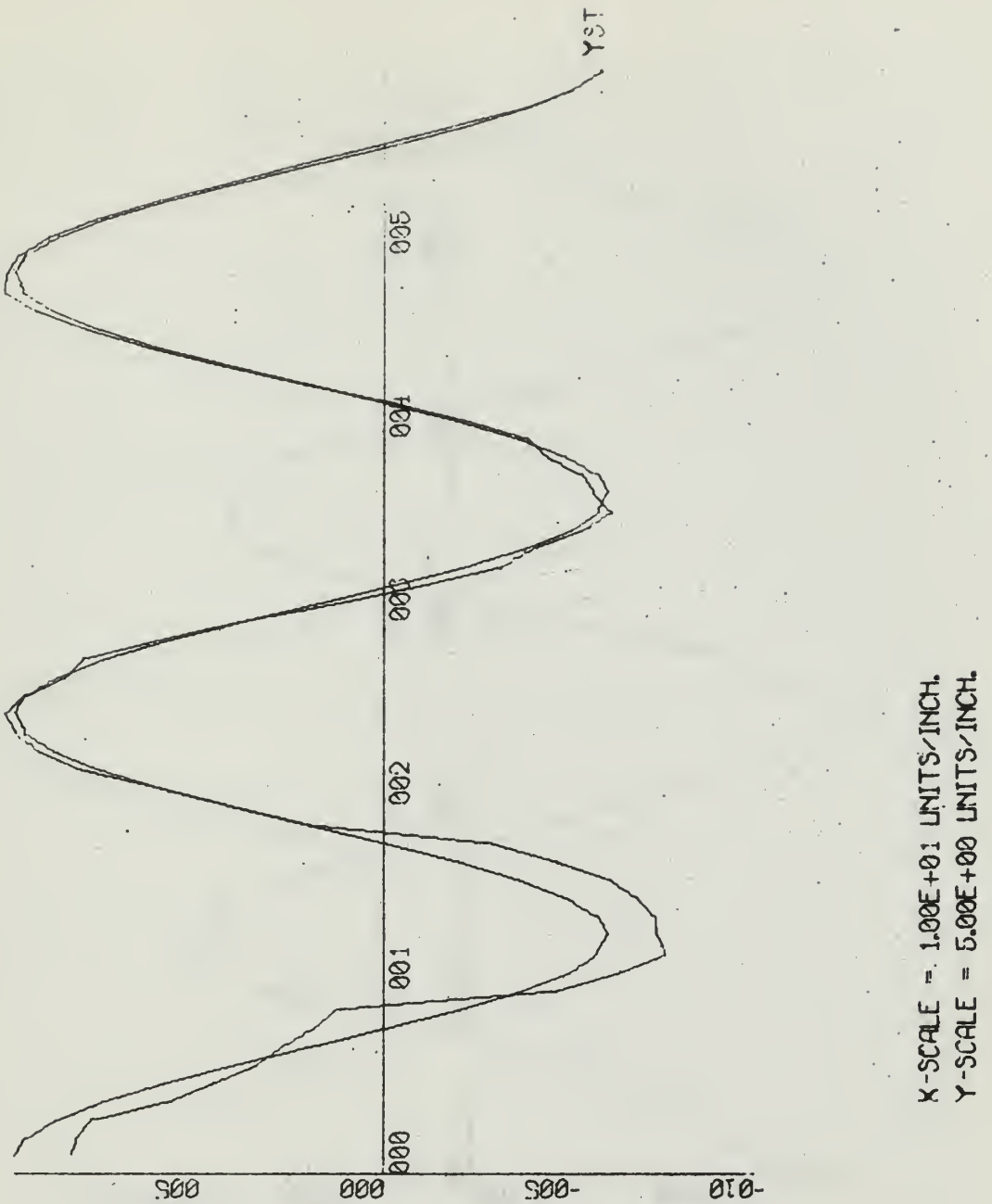
Filter computations are straightforward, and need not be elaborated upon. Results of the simulation are displayed in Figs. 2-8, 2-9 for the unexcited model, and Figs. 2-10, 2-11 for the stable model. Comparison of the resulting estimates shows that both models achieved approximately the same estimation accuracy. The simulation program listing is included in Appendix III.



X-SCALE = 1.00E+01 UNITS/INCH.
Y-SCALE = 5.00E+00 UNITS/INCH.

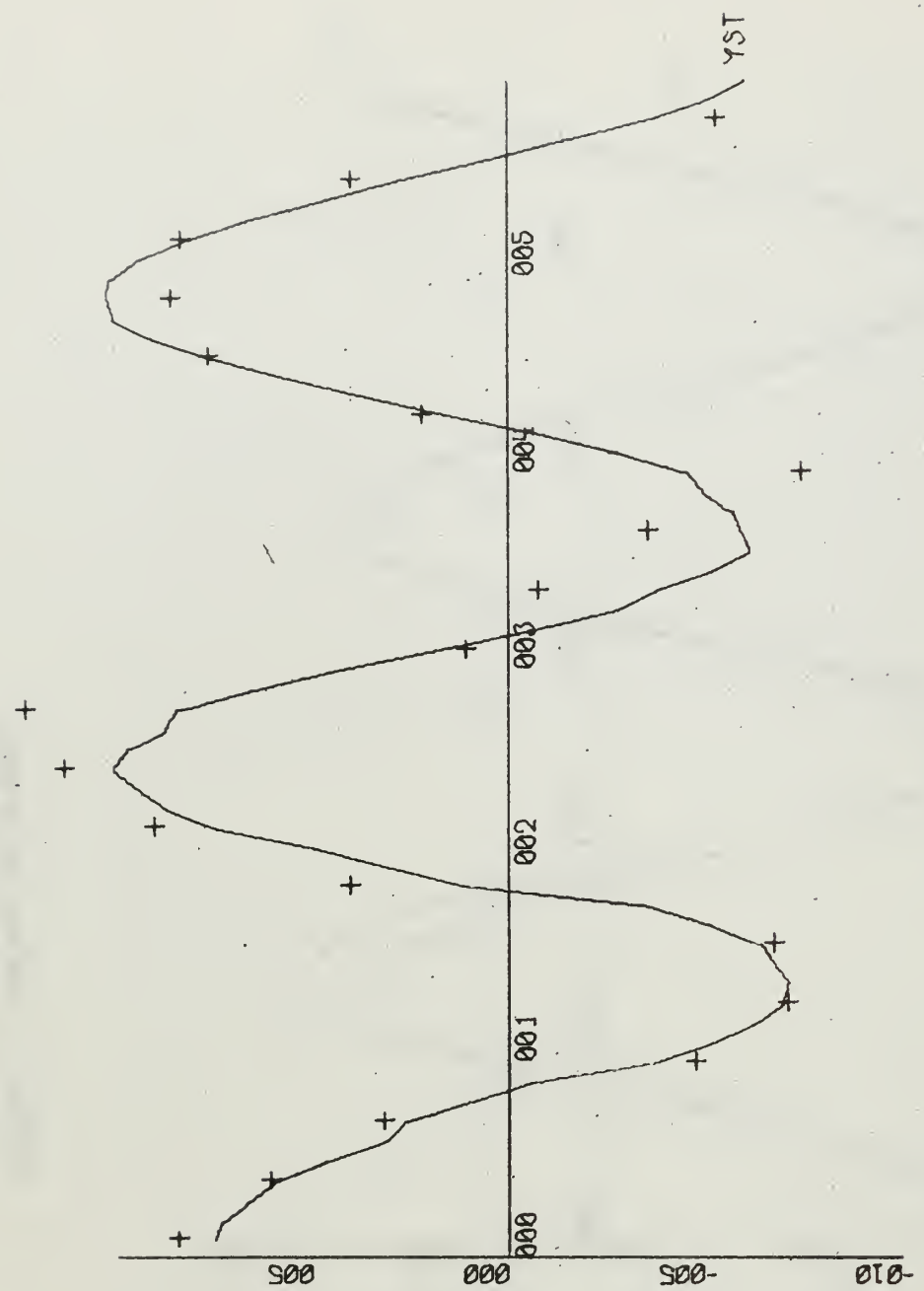
Observed and Estimated Latitude Error - Unexcited Model

Fig. 2-8



Estimated and Actual Latitude Error - Unexcited Model

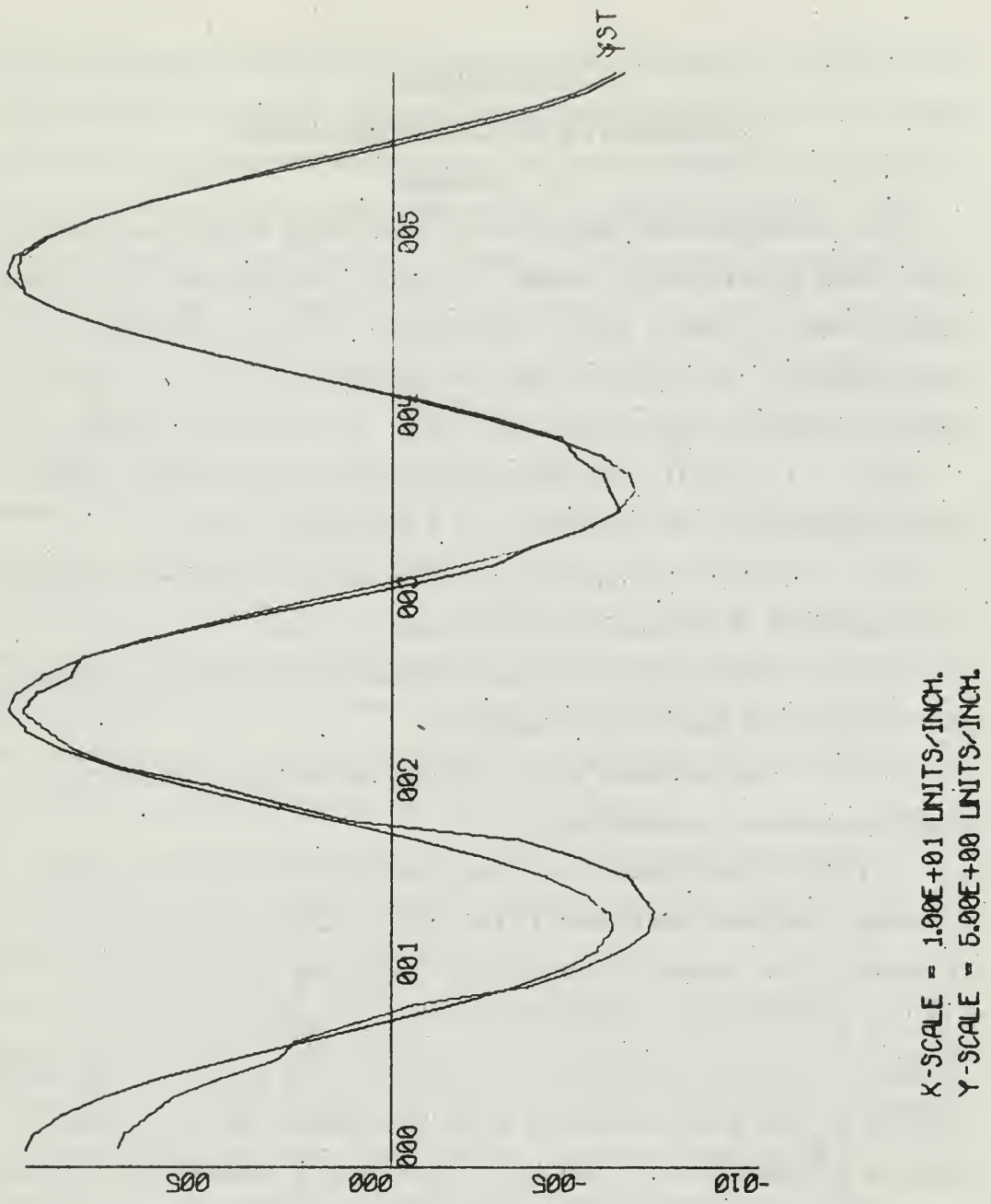
Fig. 2-9



X-SCALE = 1.00E+01 UNITS/INCH.
Y-SCALE = 5.00E+00 UNITS/INCH.

Observed and Estimated Latitude Error - Excited Model

Fig. 2-10



Estimated and Actual Latitude Error - Excited Model

Fig. 2-11

CHAPTER III

MODELS FOR RANDOM TIME SERIES

I. GENERAL

The purpose of this chapter is to treat some valid questions which arise when application of Kalman estimation techniques is considered for problems in random signal detection and pattern recognition. In such problems, the nature of possible random time series to be processed becomes quite general, and some uncertainties faced are:

(i). Is it still a justifiable assumption that the time series to be processed is the realization of a Gaussian process?

(ii). Can it be expected that an acceptable finite-parameter representation of the signal process may be found?

(iii). What finite-parameter schemes have been considered in the literature on time series analysis?

(iv). What schemes are available for obtaining the various finite-parameter representations?

In considering further the questions above, it will be helpful to review pertinent work done in the field of time series analysis. In recent years, works of Tukey [4], Grenander and Rosenblatt [11, 12], Parzen [25], and Wold [39] are representative and contain references to additional literature on the subject. The central problem in time series analysis is the following: one is allowed to observe a time series, a partial realization of a stochastic process, and on the basis of this observation is required to make statistical inference about the time series. In general, the required inference concerns the possible mechanism generating the time series or prediction of future behavior of the series.

Work done to the present in time series analysis has primarily dealt with single time series[1], resulting from stationary (and ergodic)

[1]It is only the single time series, i.e. the single-output signal generating process, which it is desired to consider here. Certain work has been done in the analysis and model determination of multivariate time series, and may be found in [3], [40], [30].

random processes with finite first and second moments. In fact, the great majority of effort in the analysis of random time series has been directed toward spectral analysis or correlation analysis, employing only the first and second moments of the generating process. This has come about for two reasons. First, it is possible to accomplish a great deal with only second order quantities, and second, the use of higher order moments introduces as yet intractable difficulties [13]. Thus the theory which has been developed has been tailored to fit the methods used. Of course in applying the theory, it is necessary to keep in mind that there are time series about whose statistical behavior only limited information is conveyed by the correlation or spectrum. Two time series may have the same autocorrelation function or power spectral density, yet possess quite different statistical properties. A classic example is provided by the so-called random telegraph signal and the output of a proper low-pass filter excited by Gaussian white noise.

From the above discussion and the fact that the Kalman technique employs linear processing, the answer to (i) is clear. As long as the processing scheme to be used employs only the first two moments of the time series to be analyzed, then without loss of generality, the subject time series may be replaced by any time series having the same autocorrelation function [6]. In particular, it may be replaced by a Gaussian time series, a sample function of a Gaussian random process, which is completely described by its first and second moments.

Remaining sections of this chapter will deal with the questions of finite parameter representations of time series and methods of estimating the parameters.

II. FUNDAMENTAL MODELS FOR RANDOM TIME SERIES

As stated above, the principal matter of concern in time series analysis is the making of inferences about the structure of a random process $\{y(t)\}$ based on observation of a partial realization of the

process, a time series $y(1), y(2), \ldots y(N)$. In general, analysis requires a mathematical model of the time series. For practical reasons, it is highly desirable that the model be of finite-parameter form, and moreover, that it be of as low an order as possible.

Since attention here is being restricted to linear processing schemes, for which as seen above, the random series may be adequately represented by its first and second order moments, use in analysis of either the power spectrum or autocorrelation function is equally valid. For investigating modeling potentialities, it turns out to be more convenient to deal with the power spectrum than the covariance sequence.[1]

In 1938, Wold [38] showed that for the discrete time case, there exists a non-decreasing bounded function $\mathfrak{J}(\omega)$, defined for $-\pi \leq \omega \leq \pi$, such that

$$R(\tau) = \int_{-\pi}^{\pi} e^{j\omega\tau} \, d\,\mathfrak{J}(\omega) \qquad \tau = 0, \pm 1, \ldots\ldots$$

The function $\mathfrak{J}(\omega)$ is called the Spectral Distribution Function of the time series. This theorem by Wold corresponds to that of Khintchine in 1933 for the continuous case, where he showed that there exists a non-decreasing bounded function $\mathfrak{J}(\omega)$, defined for $-\infty \leq \omega \leq \infty$, such that

$$R(\tau) = \int_{-\infty}^{\infty} e^{j\omega\tau} \, d\,\mathfrak{J}(\omega)$$

$\mathfrak{J}(\omega)$ can be uniquely written [5] as the sum

$$\mathfrak{J}(\omega) = \mathfrak{J}_{ac}(\omega) + \mathfrak{J}_{dd}(\omega) + \mathfrak{J}_{sc}(\omega)$$

where the three distribution functions have the following properties.

[1] The work here will deal with discrete-time random time series. Though the supporting theory has been developed for the continuous time case, it is considerably more complicated [3] than that for discrete-time sequences. Besides, for engineering purposes, useful results in the continuous case may often be had by taking discrete-time results to the limit.

$\mathfrak{J}ac(\omega)$ is absolutely continuous and is the integral of a non-negative function $F(\omega)$ called the Spectral Density Function or Power Spectral Density of the time series. The function $\mathfrak{J}d(\omega)$ is a discrete function which represents the contribution of power at discrete frequencies to the Spectral Distribution Function of the time series. It may be represented as

$$\mathfrak{J}d(\omega) = \sum_{\lambda} \Delta\, \mathfrak{J}(\omega i)$$

where

$$\Delta\, \mathfrak{J}(\omega i) = \mathfrak{J}(\omega i + 0) - \mathfrak{J}(\omega i - 0)$$

The remaining constituent function $\mathfrak{J}sc(\omega)$ is a singular continuous function, constant except on a set of Lebesgue measure zero [11]. It has been asserted [13] that this part does not appear to be meaningful observationally, and it is commonly neglected in the literature.

Having noted the elemental parts of the general Spectral Distribution Function $\mathfrak{J}(\omega)$, it will be constructive to examine models which have been proposed for time series and to consider their capacities for representing the various constituent functions.

Method of Hidden Periodicities

One of the earliest models in the history of time series analysis, the so-called scheme of hidden periodicities was introduced in 1898 by Schuster in connection with meteorological studies. The model assumes that the observed random sequence $\{y(k)\}$ may be represented as the sum of a mean value function $m(k)$ and white noise $v(k)$

$$y(k) = m(k) + v(k)$$

where the mean value function $m(k)$ represents a systematic oscillation

$$m(k) = \sum_{i=1}^{m} A_i \cos(\omega_i k + \phi_i).$$

In the model, the amplitudes A_i, angular frequencies ω_i, and

phase angles ϕ_i are constants, some of which are known and some to be estimated. The samples of white noise $v(k)$ have the properties:

$$E[v(k)] = 0 \qquad E[v(k)v(j)] = \begin{cases} \sigma^2 & k = j \\ 0 & k \neq j \end{cases}$$

Obviously, the spectral distribution function of any signal generated by such a harmonic model would correspond only to the discrete spectral distribution function above. But time series observed in nature generally possess at least a mixed spectrum [11], whose spectral distribution function contains both the discrete and absolutely continuous components. Furthermore, to a good approximation, many physically observed time series may be considered to have spectra corresponding only to the absolutely continuous spectral distribution function $\mathfrak{F}ac(\omega)$. Hence, as might now be predicted by the theory (still nonexistent at the time the scheme of hidden periodicities was introduced), such a harmonic model did not perform satisfactorily for many time series analysis problems [39].

Linear Models

Of greater practical interest are the various linear models which have been proposed. The spectra of such models correspond to absolutely continuous spectral distribution functions. This class of models includes the Autoregressive and Moving Average schemes introduced in the late 1920's by Yule and Slutzky, and the linear filter excited by white Gaussian noise used by Rice [29]. Models of the class are defined below, and techniques for estimating their parameters are considered in the next section.

Autoregressive Process

The current value of the time series is represented as the sum of a linear combination of the past n values of the time series and an uncorrelated disturbance $w(k)$, where n is the order of the process.

In difference equation form, the autoregressive process is

$$y(k) = b_1 y(k-1) + b_2 y(k-2) + \ldots b_n y(k-n) + w(k)$$

where the disturbance $w(k)$ has the properties,

$$E[w(k)] = 0 \qquad E[w(k)w(j)] = \begin{cases} \sigma^2 & k = j \\ 0 & k \neq j \end{cases}$$

A necessary and sufficient condition that the model be convergent is that all roots z_i of the polynomial

$$z^n + b_1 z^{n-1} + \ldots b_{n-1} z + b_n$$

lie within the unit circle.

Process of Moving Averages

For a model of order m, the current value of the time series is represented as a linear combination of the past m values of the uncorrelated disturbance. In equation form,

$$y(k) = a_0 w(k) + a_1 w(k-1) + \ldots a_m w(k-m)$$

where $w(k)$ is defined as for the autoregressive model above.

Linear Filter Model

In discrete-time formulation, this model takes the form

$$x(k) = \Phi x(k-1) + \Gamma w(k)$$

$$y(k) = Hx(k)$$

where

$x(k)$ = Vector of states at time k of the linear dynamic system comprising the filter.

Φ = State transition matrix of the linear dynamic system.

Γ = Vector which distributes the excitation noise across states of the filter.

H = Observation vector, which relates the scalar observable to states of the filter.

$y(k)$ = Value of the random time series at time k.

$w(k)$ = White Gaussian excitation noise as defined above.

Models a and b above, on the one hand, and model c on the other, have been employed by different groups of researchers for seemingly very different problems. Models a and b, though initially

proposed for studies of sunspot activity, have in recent years been most extensively employed in econometrics research and have to a degree come to be associated with that field of study. Conversely, the linear filter model has been studied almost exclusively by the engineering community concerned with random signal representation and stochastic control problems. It is important, however, to note their basic similarities.

The picture may be cleared by noting the capacity of each model to approximate the power spectral density of a time series to be modeled. As stated above, the spectra of linear models correspond to $\mathfrak{Jac}(\omega)$, the absolutely continuous spectral distribution function. The related power spectral density $F(\omega)$ may be approximated arbitrarily closely by a rational function in ω^2 of finite order [12]. Now in the z domain, for a spectral density which is approximately zero beyond the appropriate Nyquist frequency, there is a corresponding spectral density function rational in z, so that

$$\Lambda(z) = \frac{P(z)}{Q(z)}$$

where P and Q are polynomials.

Since the present discussion concerns discrete-time analysis, it is the relation between the linear models above and $\Lambda(z)$ which is of interest. Also, since the time series to be dealt with are real, $\Lambda(z)$ may be written as follows for z on the unit circle:

$$\Lambda(z) = \frac{P(z)}{Q(z)} = \frac{A(z)A(\frac{1}{z})}{B(z)B(\frac{1}{z})}$$

where

$$A(z) = a_0 z^m + a_1 z^{m-1} + \ldots a_m$$

$$B(z) = b_0 z^n + b_1 z^{n-1} + \ldots b_n$$

Whittle [36] has shown that a rational spectral function where A has order m and B has order n always corresponds to a process structure

$$b_0 x(k) + b_1 x(k-1) + \ldots b_n x(k-n)$$

$$= a_0 w(k) + a_1 w(k-1) + \ldots a_m w(k-m)$$

Such a model structure has been termed a mixed autoregressive-moving average model, the obvious title.

From the discussion above, relationships between the linear models mentioned and the power spectral density is clear. The linear filter model and mixed autoregressive-moving average model are equivalent and provide a rational approximation to the power spectral density. A moving average model of order m produces an m^{th} order polynomial approximation to the spectral density function, and an n^{th} order autoregressive model produces an n^{th} order inverse polynomial approximation to the spectral density function.

This section has discussed questions (ii) and (iii) raised in section I. A remaining question of key practical importance is parameter estimation for the model selected. It will be considered in brief detail next.

III. ESTIMATION OF LINEAR MODEL PARAMETERS

It has been suggested above that linear models have valuable practical utility for representing time series. Final employment in real applications, however, rests upon the ability to estimate the necessary model coefficients. The problem differs quite significantly from the problem of plant identification in control system engineering. In the present case, inputs to the filter are known only in statistical terms, and as a matter of fact, are not physical quantities at all, but are fictitious.

In the section above, the linear filter and mixed autoregressive-moving average models were asserted to be equivalent. It is therefore helpful to express the linear filter in such a form that correspondence between parameters of the two models may be easily shown. The "standard canonical form" of [21] is such a form. It is obtained

through a linear transformation of the state variables, and involves no alteration of the filter input-output relation. The observability requirement for such a transformation is no restriction in the present situation. In the "standard canonical form", the three matrices describing the filter input-output relation have the form:

$$\Gamma = \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \cdot \\ \cdot \\ \cdot \\ \gamma_n \end{bmatrix} \qquad \Phi = \left[\begin{array}{c|ccc} 0 & & I & \\ & & & \\ \hline -b_n & \cdot & \cdots & -b_2 \; -b_1 \end{array}\right] \qquad H = \left[\begin{array}{c} 1 \\ \hline \\ \\ 0 \\ \\ \end{array}\right]$$

After normalizing coefficients of the mixed autoregressive-moving average model to obtain b_0 equal unity, remaining autoregressive coefficients b_i directly equate to elements of the Φ matrix bottom row as shown. Equation of moving average coefficients a_i with filter parameters is not as direct and is given below.

$$\begin{bmatrix} a_0 \\ a_1 \\ \cdot \\ \cdot \\ \cdot \\ a_m \end{bmatrix} = \begin{bmatrix} 1 & & & 0 \\ b_1 & 1 & & \\ \cdot & & & \\ \cdot & & & \\ \cdot & & & \\ b_{n-1} & \cdots & b_1 & 1 \end{bmatrix} \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \cdot \\ \cdot \\ \cdot \\ \gamma_n \end{bmatrix} \qquad (m=n-1)$$

The relation above may be obtained either by generalizing from low dimensional systems or deriving directly for the general case [14].

No general techniques presently exist for estimating the parameters a_i and b_j of a linear filter in a straightforward manner from observed data. However, if the time series can be considered to be sufficiently represented by a moving average model, then the following relations may be solved to obtain the coefficients of the

polynomial $A(z)$.

$$\begin{bmatrix} a_0 a_1 & \cdots & \cdots & a_m \\ 0\ a_0 & & & a_{m-1} \\ 0 & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & a_0 a_1 & \\ 0 & & 0\ a_0 & \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \cdot \\ \cdot \\ \cdot \\ a_m \end{bmatrix} = \begin{bmatrix} r(0) \\ r(1) \\ \cdot \\ \cdot \\ \cdot \\ r(m) \end{bmatrix}$$

where $r(\tau)$ are values of the observed autocorrelation function of lag τ. As it happens, there are 2^m possible ways in which zeros of the product $A(z)A(\frac{1}{z})$ may be assigned to A to obtain the same power spectral density (and autocorrelation function). Hence there is an indeterminacy of order 2^m in the resulting coefficients of $A(z)$. This indeterminacy may be resolved by using the fact that the time series is real and by forcing all zeros of $A(z)$ to lie on or within the unit circle. The net effort, however, has been sufficient to discourage use of moving average models of very high order. It is a useful observation to note that the autocorrelation function of a series generated by a moving average model is zero for lags greater than m.

Estimation of parameters for an autoregressive model is more straightforward. The problem is to obtain a least squares fit for the coefficients b_i in the relation

$$\sum_{i=0}^{n} b_i\, y(k-i) = w(k) \qquad b_0 = 1, \quad k = 1, 2, \ldots N$$

subject to the restriction that the roots of $B(z)$ should fall within the unit circle. The results of such an approach are the consistent set of linear equations [13] ($b_0 = 1$), where the $r(\tau)$ are values of the observed correlation function of lag τ.

$$\begin{bmatrix} r(0) & r(1) & \cdots & r(n-1) \\ r(1) & r(0) & & \\ \vdots & & & \\ & & r(0) & r(1) \\ r(n-1) & \cdots & r(1) & r(0) \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b \end{bmatrix} = - \begin{bmatrix} r(1) \\ \vdots \\ r(n) \end{bmatrix}$$

An alternate approach, a recursive approach, is also possible for estimating autoregressive model parameters. Least squares estimation problems with uncorrelated residuals can be conveniently handled within the framework of the Kalman filter [10]. Such an approach has been employed by Ho and Lee [16] to estimate the coefficients of $B(z)$. From the autoregressive model definition,

$$y(k) = \phi_1 y(k-1) + \phi_2 y(k-2) + \ldots . \phi_n y(k-n) + v(k)$$

where

$$\phi_1 = -b_1 \, , \quad \phi_2 = -b_2 \quad \text{etc}$$

Let

$$\underline{\phi}^T = (\phi_1 \phi_2 \ldots \phi_n)$$

$$\underline{y}_k = (y(k-1) \quad y(k-2) \ldots . \quad y(k-n))$$

Then

$$y(k) = \underline{y}_k \underline{\phi} + v(k)$$

For the corresponding Kalman filter, the required quantities are:

$$\Gamma Q \Gamma^T = 0 \qquad \underline{x} = \phi \qquad z(k) = y(k)$$

$$\Phi = I \qquad H = \underline{h} = \underline{y}_k \qquad R = r = E[v(k)^2]$$

At a glance, it is clear that the computations required are simple. Since the filter output is a scalar, no matrix inversion is required. Further, gain computations are simplified here since $P(k/k-1) = P(k-1/k-1)$. It may first appear that ignorance of r will present difficulty. However, normalizing the weighting computations with

respect to r and employing the suboptimal but practically useful device [21] of a very large P(1/0) avoids the problem.

The recursive approach is quite flexible. If it is desired to estimate the parameters of the best autoregressive model for the observed time series, each sample in turn is processed. On the other hand, if it is required to estimate the denominator coefficients of a linear filter having a numerator of order m, then processing only every m+1 samples guarantees uncorrelated residuals (recalling that correlation in a moving average model is nonzero only over m lags). Then the numerator coefficients may be obtained by processing the residuals left after treating the observations with the denominator in an autoregressive fashion.

The approach of Ho and Lee has been used in experiments to estimate the model coefficients for an autoregressive representation of a random signal. Results are given in the next section.

IV. EXAMPLES OF MODEL DETERMINATION

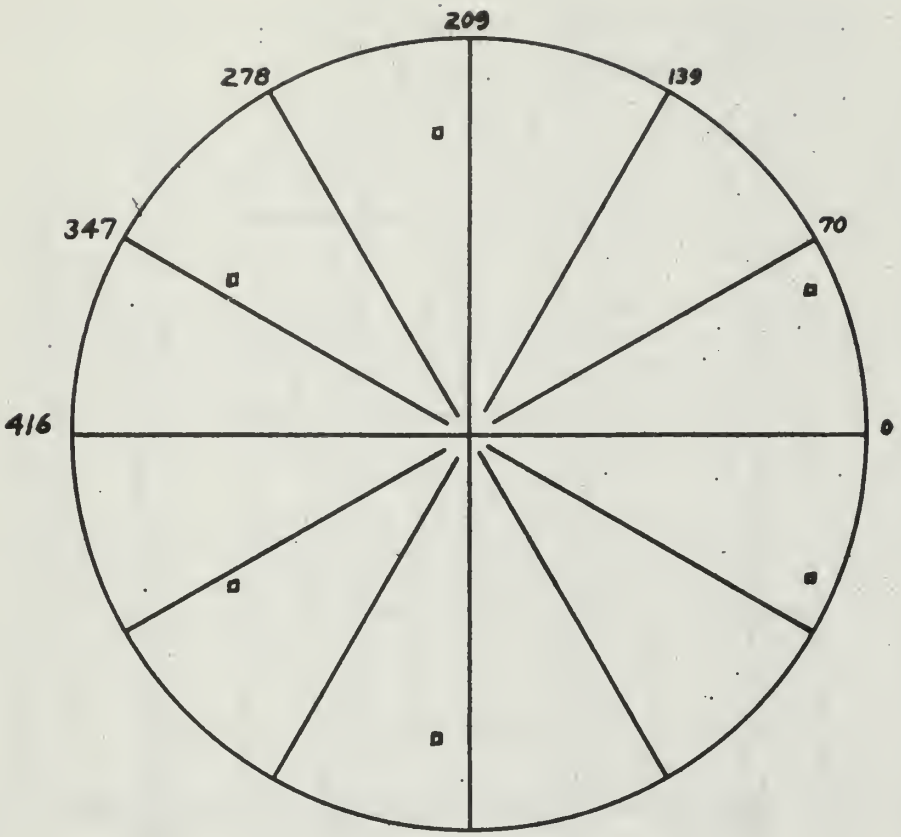
Various facets of the discussion above may be made clearer by an illustrative example. Recursive parameter estimation was employed to obtain an autoregressive model for a contrived laboratory signal. The model to be found was arbitrarily chosen to be of sixth order.

The contrived signal consisted of the additive combination of a 50 cycle sine wave and the output of a laboratory random noise generator. Originally generated in analog form, the signal was sampled by an analog-to-digital converter under control of a CDC 160 computer, and the digital samples were stored on magnetic tape in 4000-sample blocks.[1] Since initial analysis disclosed little noise energy above approximately 400cps, the sampling interval was effectively increased from the original 400 μs to 1200 μs by using only every third sample, giving a Nyquist frequency of 416 cps. Of course, the resulting sample size used in obtaining the model was

[1]The very useful programs used to accomplish the digitizing/recording operation and necessary subsequent unpacking are due to N. Barrett [2].

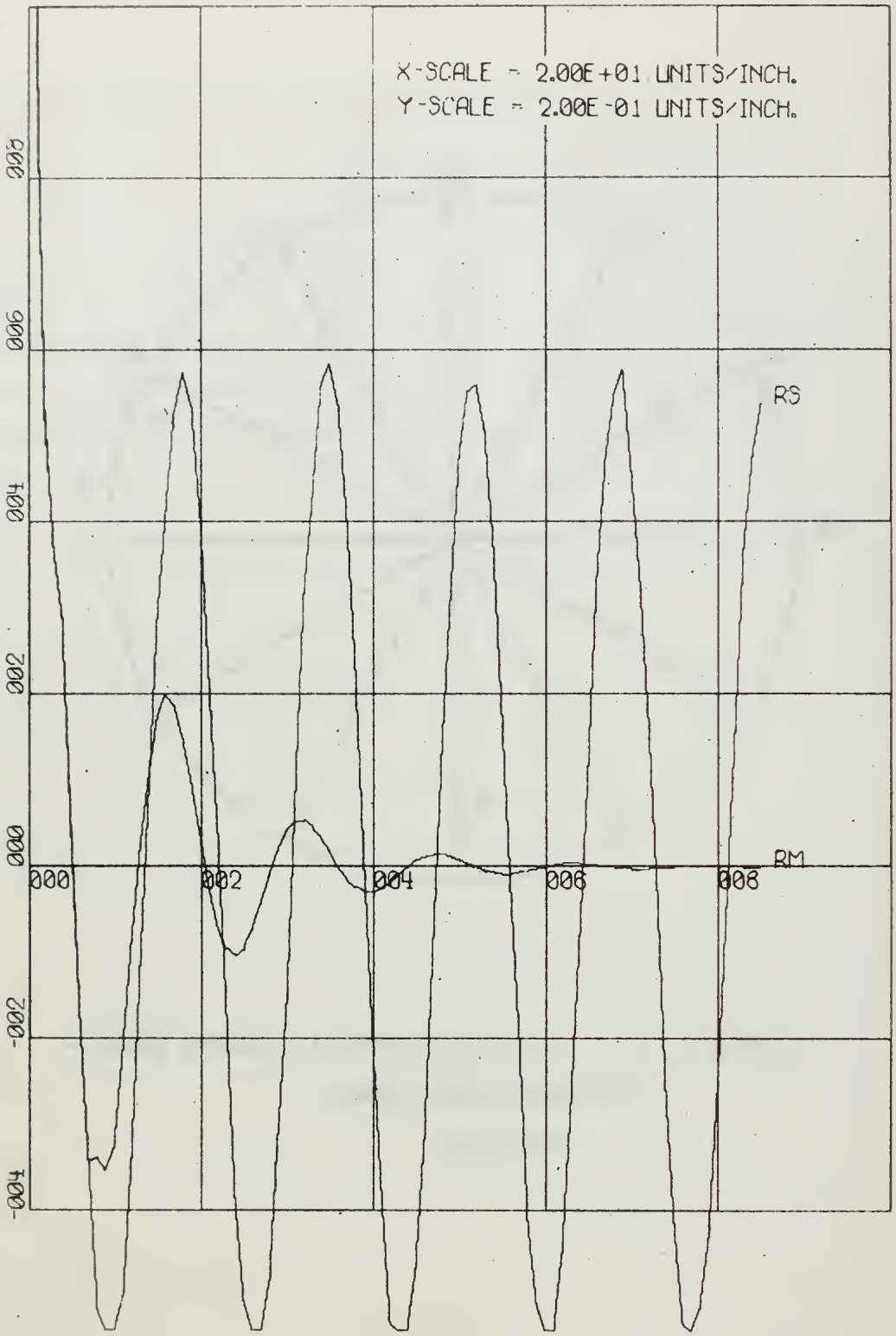
reduced to a third of the original 4000 samples.

Two runs were made, using signal-to-noise ratios of approximately 0 db and -10 db. Results are displayed in Figs. 3-1 to 3-6. Plots of z-plane poles of the resulting models are given in Figs. 3-1 and 3-4. The remaining figures for each run contain comparative plots of autocorrelation functions and power spectral densities derived from the signal and model. A visual indication of model efficiency is provided by the comparative power spectral density plot.



Model Poles - 50 cps Sine Wave in Random Noise -
0 db Signal/Noise Ratio

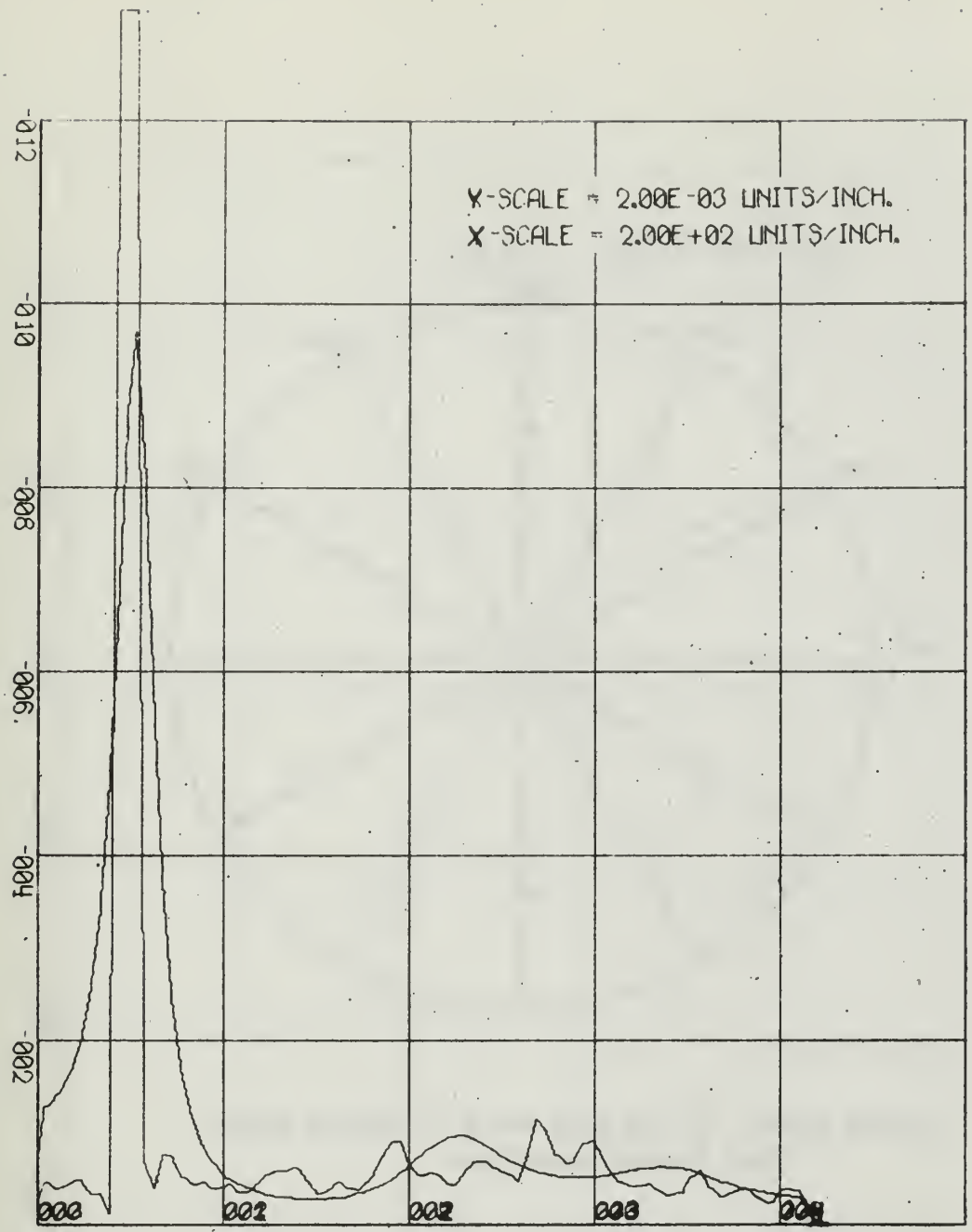
Fig. 3-1



Autocorrelation Functions Computed from Data and Model

S/N = 0 db

Fig. 3-2



Power Spectral Densities Computed from Data and Model

S/N = 0 db

Fig. 3-3

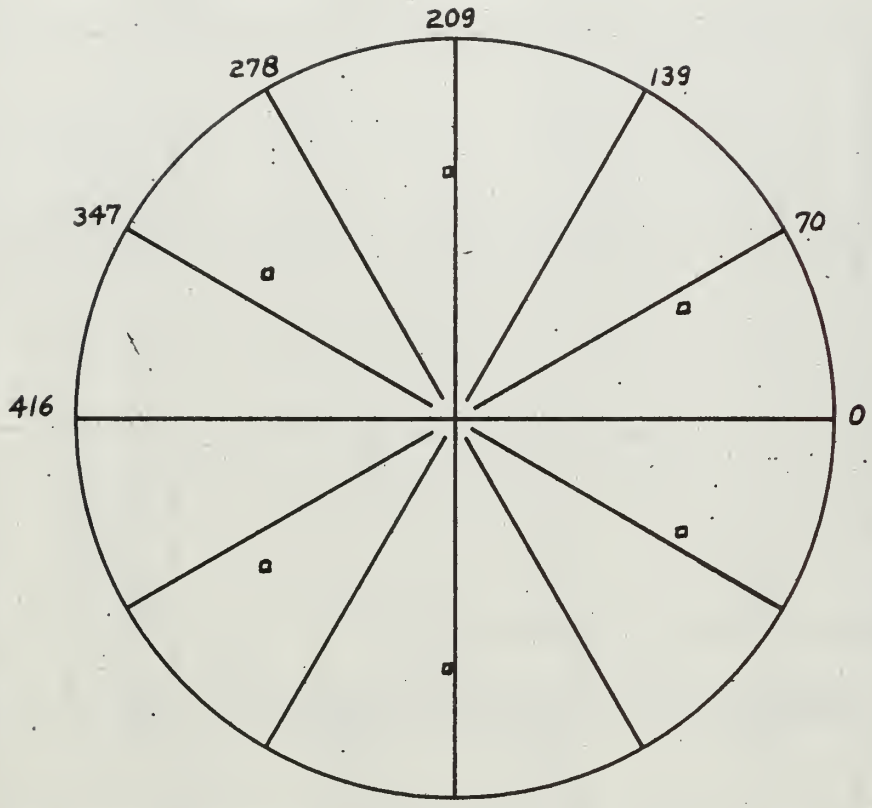


Fig. 3-4 Model Poles - 50 cps Sine Wave in Random Noise - -10 db Signal/Noise Ratio

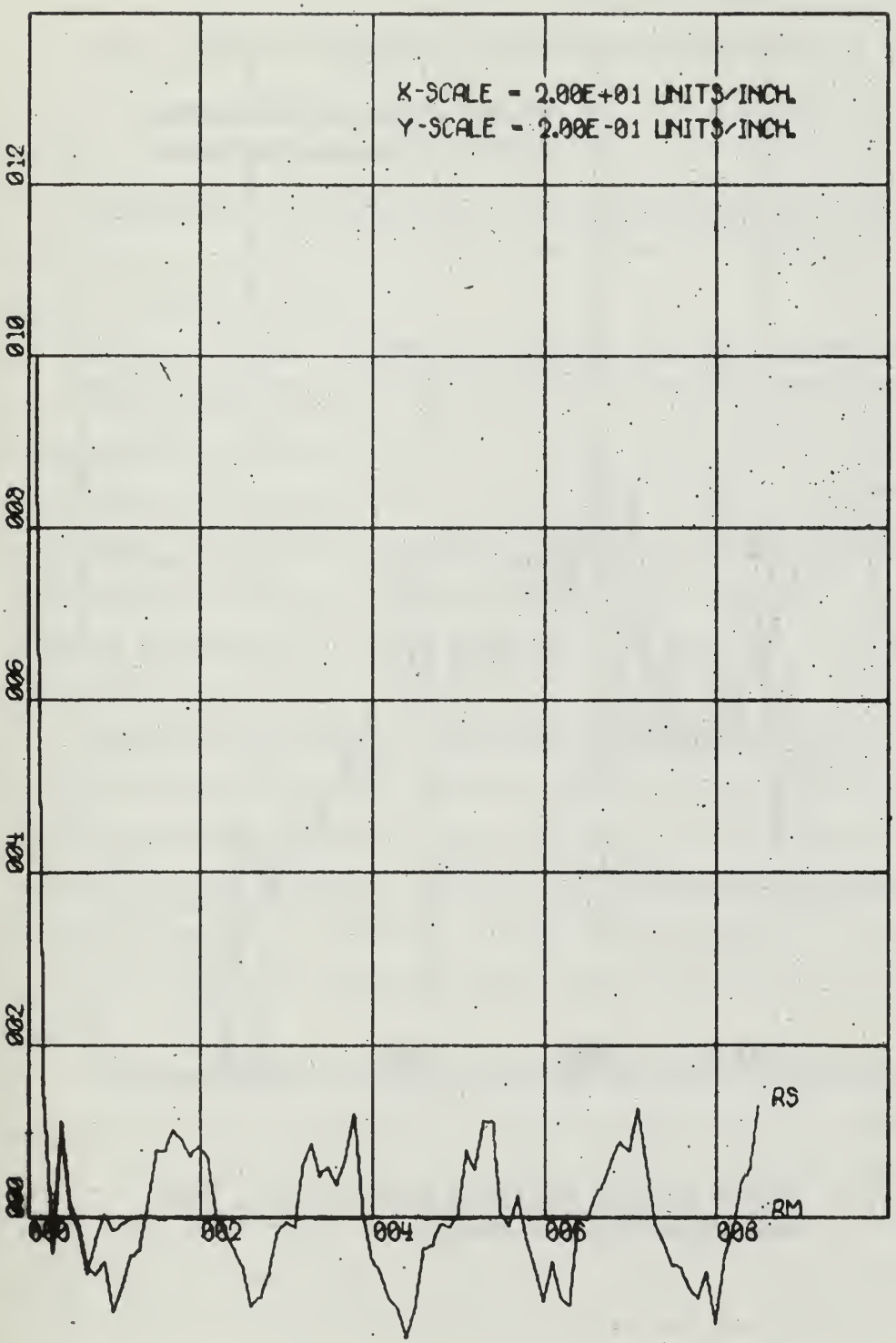


Fig. 3-5 Autocorrelation Functions Computed from Data and Model - S/N = -10 db

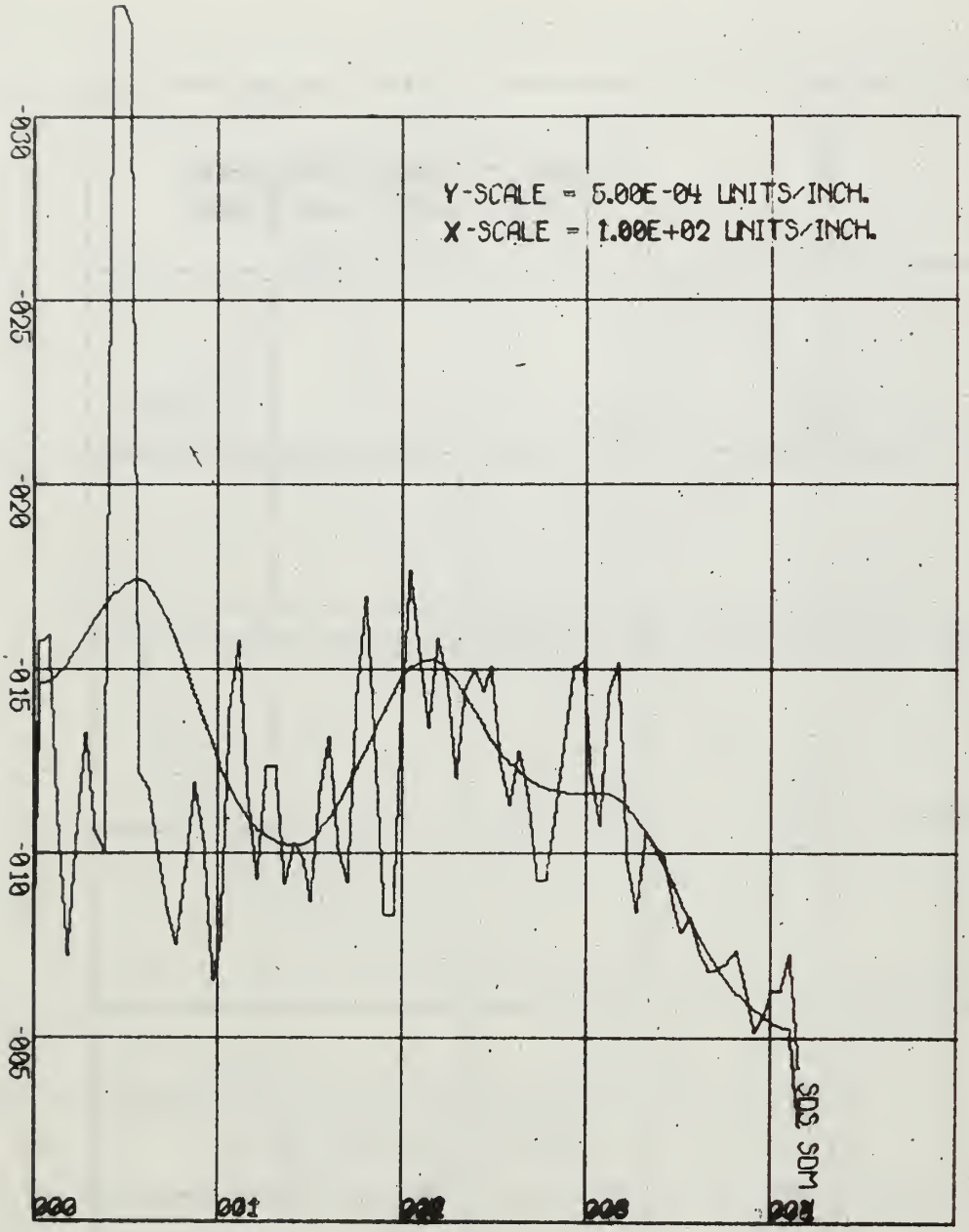


Fig. 3-6 Power Spectral Densities Computed from Data and Model - S/N = -10 db

CHAPTER IV

DETECTION OF RANDOM SIGNALS IN NOISE

The problems of detecting both deterministic[1] and random signals in the presence of additive Gaussian noise have received considerable attention in the literature [14], [22], [23], [34], [37]. It is well known that for the former case, the optimum receiver is a cross correlation or matched filter, which correlates the input with a replica of the transmitted signal.

Random signals considered previously, and to be considered here, have been assumed to be of the somewhat restrictive but tractable Gaussian form with zero mean and known autocorrelation. (This assumption will be discussed in greater detail below.) Kailath [18] has shown that the optimum receiver for such Gaussian signals in Gaussian noise forms a cross correlation of the input and a least squares estimate of the signal based on input received in the interval under scrutiny.

The results obtained in [18] suffer practical drawbacks, however. Critical computational difficulties are faced in obtaining the least squares estimate when the number of samples considered is large, as it is required to invert a matrix of the same dimension as the number of samples. Moreover, statistical descriptions of possible signals, which were assumed known, are in practice not easy to obtain.

The present work approaches this problem in a manner somewhat similar to that of Weaver [35], and employs the results of Schweppe [31]. The serious computational difficulties noted above are avoided. Specifically, estimation techniques developed in recent years by Kalman and others [19], [20], [26], [27] are employed. Furthermore, in view of the similarities between correlation signal

[1] In the discussion to follow, a deterministic signal will be defined as one whose form is completely known at the receiver, and a random signal will be one whose description is known at the receiver only in statistical terms.

detection and linear discrimination methods in pattern recognition, it is felt that the approach to be followed here may constitute a new and more powerful approach to certain pattern recognition problems.

For simpler and clearer presentation of results, the analysis to follow will treat the discrete, or sampled, situation. However, the techniques to be employed are equally applicable to continuous processing.

I. OUTLINE OF THE PROBLEM

The problem to be considered is the following: a signal consisting of a finite number N of samples of a Gaussian process $y(t)$ is received during a limited time interval T in the presence of additive Gaussian noise. The additive noise and signal process are assumed independent. In practice, real time processing is necessary, and it is required to announce at the end of the interval whether or not the signal $y(t)$ has occurred. It will be seen later that the present limitation to a single signal is not restrictive. The same approach applies to the problem of detecting and discriminating members of a finite set $\{ y^m(t) \}$ of signals.

A word about the assumption of Gaussian signals is in order. Aside from transmitted signals whose forms are simply not known at the receiver except in terms of their first and second order statistics, it happens that the scatter-multipath structure of ionospheric propagation perturbs transmissions into Gaussian signals [27]. The same situation is produced in radar and sonar by clutter and reverberation resulting from the effects of a multitude of small random scatterers.

II. CORRELATION DETECTION OF DETERMINISTIC SIGNALS

It will be convenient to first consider the problem of correlation detection of deterministic signals in order to establish notation conventions, more clearly outline the problem, etc.

From the problem statement above, the task at hand is clear. The received signal, a mixture of the transmitted signal and additive

white noise, must be processed to yield an indication of which transmitted signal in a finite set of possible signals $\{y^m(t)\}$ was received. It is, of course, desired that the indication be as reliable as possible under the circumstances.

The essential feature of the problem is extraction of information about the transmitted signal from the mixture of transmitted signal and additive noise. Information concerning the transmitted signal is clearly more germane to the issue than is determination of signal-to-noise ratios [37], though the information sought and signal-to-noise ratio may often be monotonically related. Possessing as much information about the transmitted signal as it is possible to isolate from the received signal available for processing, one may proceed with the required decisions. The decision making will commonly also consider other factors, such as relative penalties for incorrect decisions, any a priori information, etc.

As mentioned, only a mixture, here called $z(t)$, of the transmitted signal $y^i(t)$ and noise $v(t)$ is available for processing, i.e. $z(t) = y^i(t) + v(t)$. The noise $v(t)$ is assumed to be white Gaussian noise with mean zero and known variance r. Since the discrete case is being considered here, what is available for processing is a sequence of samples $z(k)$ of $z(t)$, where $z(k) = y^i(k) + v(k)$.

At this point, it will simplify notation if we define vectors y^i and z, where

$$y^i = y^i(1), y^i(2) \ldots\ldots y^i(N)$$

$$z = z(1), z(2) \ldots\ldots z(N)$$

From the problem statement, it can be seen that what must be performed is a multiple-alternative hypothesis test [22]. Stated another way, as viewed by the receiver, y is a discrete random variable having the possible values y^i $i = 1, m$. All information at the receiver about the transmitted signal y is contained in the conditional probabilities of y given the observations z, $p(y^i/z)$ [37]. It is these conditional probabilities which will be considered further.

Remaining aspects of the decision-making problem (relative costs of errors, etc.) depend upon the specific situation.

The conditional probabilities $p(y^i/z)$ may be stated in terms of given or easily determined quantities through use of Bayes Formula [17],

$$p(y^i/z) = \frac{p(z/y^i)p(y^i)}{p(z)}$$

On the right hand side, $p(y^i)$ is the a priori probability of signal y^i (assumed given). $p(z)$ is not a function of the transmitted signal y^i and may be considered a normalizing factor. The remaining quantity, $p(z/y^i)$ is a function both of the transmitted and received signals, and must be considered in detail.

Expanding the notation,

$$p(z/y^i) = p[z(1), z(2) \ldots .z(N)/y^i(1), y^i(2) \ldots .y\ (N)]$$

Due to whiteness of the additive noise, consecutive samples of $n(t)$ are uncorrelated. Hence

$$z(1)/y^i(1), y^i(2) \ldots y^i(N) = z(1)/y^i(1) \sim N[y^i(1), r]$$

$$z(2)/y^i(1), y^i(2) \ldots .y^i(N) = z(2)/y^i(2) \sim N[y^i(2), r]$$

etc.

Furthermore, $z(j)/y^i(j)$ and $z(k)/y^i(k)$ are independent random variables for all $j \neq k$. Therefore,

$$p[z(1), z(2) \ldots .z(N)/y^i(1), y^i(2) \ldots y^i(N)]$$
$$= \prod_{j=1}^{N} p[z(j)/y^i(j)]$$

In more compact form,

$$p(z/y^i) = (2\pi r)^{-\frac{N}{2}} \exp -\frac{1}{2r} \sum_{j=1}^{N} [z(j) - y^i(j)]^2$$
$$= C \exp -\frac{1}{2r} \left[\sum_{j=1}^{N} z(j)^2 + \sum_{j=1}^{N} y^i(j)^2 - 2 \sum_{j=1}^{N} z(j)y^i(j) \right]$$

Inside the brackets, the first term represents received signal energy, a factor which will cancel in later comparisons of conditional probabilities. It can therefore be incorporated into C. The second term represents energy of the transmitted signal y^i. If all the possible transmitted signals have been normalized so as to possess the same energy, this term can also be incorporated into C. Otherwise, it may be carried along as an additional constant C^i. The situation becomes,

$$p(z/y^i) = CC^i \exp \frac{1}{r} \sum_{j=1}^{N} z(j)y^i(j)$$

It is seen that the optimum receiver, which furnishes the conditional probabilities $p(y^i/z)$ to the decision making processor, forms a cross correlation of the received signal and the transmitted signal and uses the result in computing the desired conditional probability. The resulting optimum receiver structure is shown in Fig. 4-1.

The subject of correlation detection is far from exhausted. In particular, the decision philosophy to be employed and details thereof have been omitted. The purpose of this section has been to establish a frame of reference to be extended below in discussion of random signal detection. Further discussion of the problem of correlation detection, as well as references to the literature on this subject, may be found in [22], [37].

While the discussion above has followed the decision-theoretically optimum Bayesian approach to signal detection, situations exist where no meaningful values for the required a priori probabilities can be obtained. It is customary in such situations to employ the philosophy of Maximum Likelihood suggested by R. A. Fisher. Evaluation of the Likelihood Function, defined as the same conditional probability density function $p(z/y^i)$ considered above, is required. The resulting quantities are weighted by relative costs of misclassification and compared, yielding a decision that that signal

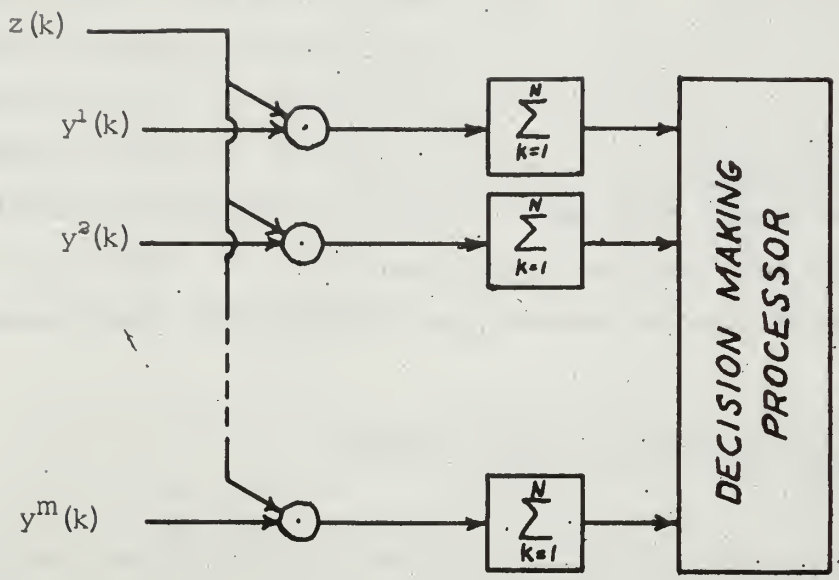


Fig. 4-1 Optimum Receiver for Deterministic Signals

y^i is present which corresponds to the maximum Likelihood Function [8]. The essential point is that the Likelihood Function must be evaluated in either approach. Hence one may approach the task with relatively more confidence that efforts are well directed.

III. RANDOM SIGNAL DETECTION

In this section, the ideas developed above will be extended to handle detection of Gaussian signals in white Gaussian noise. First the approach of Kailath [18] will be discussed and its computational difficulties noted. Then application of recently developed filtering and smoothing techniques to this problem will be considered.

Here the signal is assumed to comprise a sequence of N samples from a Gaussian process whose mean value function and covariance function are known. Hence, each signal vector y^i possesses a multivariate normal distribution with mean vector $\mu = 0$ and covariance matrix K_y^i.

$$K_y^i = E(y^i y^T) = (R_{ij}^i) \qquad i, j = 1, 2 \ldots . N$$

Just as in detection of deterministic signals above, the problem reduces to evaluation of the Likelihood Function $p(z/y^i)$. Since the signal and additive noise are assumed statistically independent, the received signal covariance is

$$K_z^i = K_y^i + R \qquad \text{where } R = rI$$

The Likelihood Function is

$$p(z/y^i) = (2\pi)^{-\frac{N}{2}} |K_z^i|^{-\frac{1}{2}} \exp -\frac{1}{2}[z^T (K_z^i)^{-1} z]$$

It may be expanded using a matrix inversion lemma arising from the Frobenius-Schur Relation [7]. (Since only a single signal is being considered for the moment, notation will be simplified by omitting the superscript temporarily.)

$$K_z^{-1} = R^{-1} - R^{-1}(R^{-1} + K_y^{-1})^{-1} R^{-1}$$

Thus,

$$p(z/y) = C \exp -\frac{1}{2} \left\{ z^T R^{-1} z \; - \; z^T \left[R^{-1} \left(R^{-1} + K_y^{-1} \right)^{-1} R^{-T} \right] z \right\}$$

The first term in the exponent does not involve the signal, hence will cancel in the decision process, and may be carried as a constant C, leaving

$$p(z/y) = CC^i \exp \frac{1}{2} \left\{ z^T \left[R^{-1} \left(R^{-1} + K_y^{-1} \right)^{-1} R^{-1} \right] z \right\}$$

Now

$$\begin{aligned} \left(R^{-1} + K_y^{-1} \right)^{-1} R^{-1} &= \left(I + R K_y^{-1} \right)^{-1} \\ &= K_y \left(K_y + R \right)^{-1} \\ &= K_y K_z^{-1} = A \end{aligned}$$

It is easily shown [18], and included below, that A is the weighting matrix yielding the least mean squared error estimate vector $\hat{y}$ of y given the observation vector z.

What is required is the matrix A such that if

$$\hat{y} = Az$$

the mean squared error between $\hat{y}$ and y will be minimized. Error in estimating the i the component of y is

$$e_i = y_i - \sum_{j=1}^{N} a_{ij} \; z_j$$

Hence,

$$\begin{aligned} \overline{e_i^2} &= \overline{y_i^2} - 2 \sum_{j=1}^{N} a_{ij} \; \overline{z_j y_i} + \sum_{j=1}^{N} \sum_{k=1}^{N} a_{ij} \; a_{ik} \; \overline{z_j z_k} \\ &= R_{yy}(0) - 2 \sum_{j=1}^{N} a_{ij} \; R_{yz}(i,j) + \sum_{j=1}^{N} \sum_{k=1}^{N} a_{ij} \, a_{ik} \, R_{zz}(j,k) \end{aligned}$$

For a minimum,

$$\frac{\partial \overline{e_i^2}}{\partial a_{ij}} = 0 \qquad j = 1, \ldots . N$$

Thus

$$R_{yy}(i,j) = \sum_{k=1}^{N} a_{ik} R_{zz}(j,k)$$

(Note that $R_{yz}(\tau) = R_{yy}(\tau)$.) In matrix form,

$$K_y = AK_z \quad \Longrightarrow \quad A = K_y K_z^{-1}$$

Having computed A, the Likelihood Function to be determined takes the form,

$$p(z/y) = CC^i \exp \frac{1}{2r} (z^T Az)$$

$$= CC^i \exp \frac{1}{2r} (z^T \hat{y})$$

It is seen that the optimum receiver for detecting Gaussian signals forms a cross correlation of the received signal and a least-squares estimate of the Gaussian signal given the received signal. As in the case of deterministic signal detection, this cross correlation is used to evaluate logarithms of the conditional probabilities $p(z/y^i)$ $i = 1, \ldots .m$, which are in turn furnished to the decision making processor. The resulting receiver structure is shown in Fig. 4-2.

Recalling that the covariance matrices K_y and K_z are of dimension NxN, where N is the number of samples to be processed, the computational difficulties in obtaining $A = K_y K_z^{-1}$ are evident. Using the fact that K_z is a Toeplitz matrix, Trench [33] has developed an algorithm for obtaining the inverse requiring effort proportional only to the square of the matrix dimension, vice its cube. Thus the problem is reduced in magnitude, but remains quite a task for even moderately large sample sizes, say 200. Unfortunately, an even worse problem must still be faced. In the present case of random signals, K_z is a function of K_y^i. The quantity which was carried as a constant C (since it would later cancel) in the Likelihood Functions of deterministic signals, must now be evaluated. Evaluation of the determinant of K_z is involved. Even with exploitation

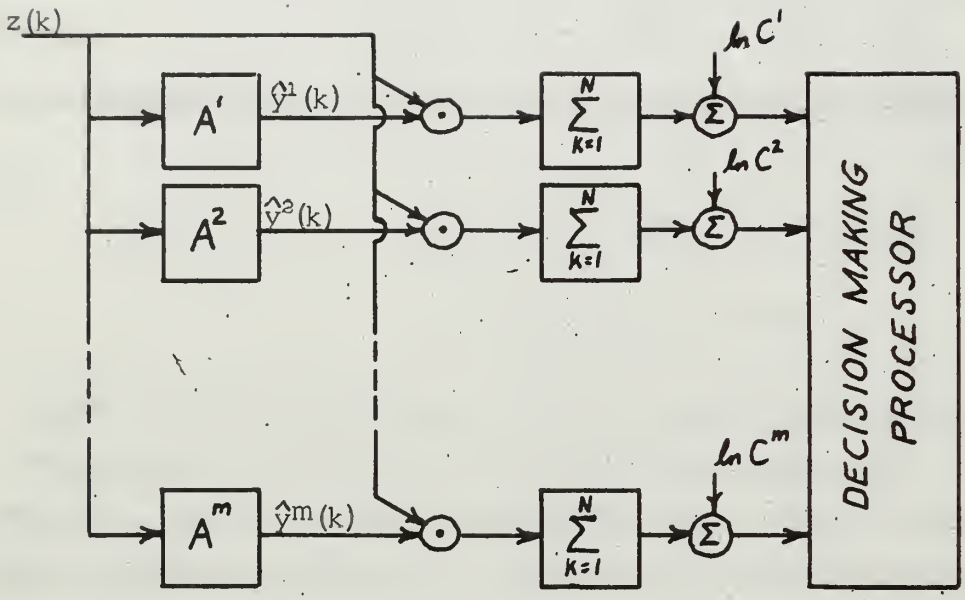


Fig. 4-2 Optimum Receiver for Gaussian Signals

of the Toeplitz form of K_z, the problem remains a considerable one.

The signal above was assumed to consist of a sequence of samples from a Gaussian random process with known mean (zero) and known covariance function. Description of the Gaussian process in an equivalent but different manner will permit solving the problem while avoiding the difficulties above. As discussed in Chapter III, the Gaussian process may be represented to any desired degree of approximation as the output of a linear dynamical system excited by white Gaussian noise.

Statistical description of the Gaussian message process will then be in terms of the mean (zero) and covariance q of the excitation noise, and in terms of matrices Γ, Φ, and H defining the input-output relation of the linear dynamical system. As before, this statistical description of the message process is assumed known.

An alternate approach to the random signal detection problem may be pursued with the message process description above using the results of Schweppe [31]. It involves developing a difference equation for the Likelihood Function (or rather its logarithm), from which the complete Likelihood Function, not just its exponent, may be determined recursively as signal samples are received. Both the problems of inverting the NxN matrix K_z and evaluating its determinant are avoided.

Let

$$L(y^i) \triangleq p(z/y^i)$$

Then

$$-2 \ln L_N(y^i) = \ln (2\pi)^N |K_z| + z^T K_z^{-1} z$$

Now,

$$L_k(y^i) = p[z(1) \ldots .z(k)/y^i]$$

$$= p[z(1) \ldots .z(k-1)/y^i] p[z(k)/z(1) \ldots .z(k-1), y^i]$$

Hence,

$$\ln L_k(y^i) - \ln L_{k-1}(y^i) = \ln p[z(k)/z(1) \ldots z(k-1), y^i]$$

The conditional probability density function

$$p[z(k)/z(1) \ldots .z(k-1), y^i]$$

must be considered further. Under the hypothesis that the transmitted signal is y^i, the matrices describing its statistical characteristics are Γ^i, Φ^i, and H^i. A Kalman filter using these matrices and operating on the received signal data computes values $x(k/k-1)$ and $P(k/k-1)$ which define the conditional distribution

$$p(x(k)/z(1) \ldots .z(k-1), y^i)$$

$$= C \exp -\frac{1}{2}[(x(k) - x(k/k-1))^T P(k/k-1)^{-1} (x(k)-x(k/k-1))]$$

From this, it follows simply that

$$p[z(k)/z(1) \ldots .z(k-1), y^i]$$

$$= (2\pi)^{-\frac{1}{2}}[HP(k/k-1)H^T + r]^{-\frac{1}{2}} \exp - \frac{[z(k) - z(k/k-1)]^2}{2(HP(k/k-1)H^T + r)}$$

Defining

$$\tilde{z}(k) \triangleq z(k) - z(k/k-1)$$

yields

$$-2 \ln p[z(k)/z(1) \ldots .z(k-1), y^i]$$

$$= \ln 2\pi[HP(k/k-1)H^T + r] + \frac{\tilde{z}(k)^2}{[HP(k/k-1)H^T + r]}$$

A difference equation in $-2 \ln L_k(y^i)$ has now been determined. The original requirement was to evaluate $L_N(y^i)$, or, equivalently, $-2 \ln L_N(y^i)$. Hence, the differences are simply summed, $k = 1, 2, \ldots N$. The result is

$$-2 \ln L_N(y^i) = \ln (2\pi)^N |K_z| + z^T K_z^{-1} z$$

$$= \sum_{k=1}^{N} \ln 2\pi [HP(k/k-1)H^T + r] + \sum_{k=1}^{N} \frac{\tilde{z}(k)^2}{[HP(k/k-1)H^T + r]}$$

It can be seen that $-2 \ln L_N(y^i)$ has been built up from sums of scalar quantities. Required quantities $\tilde{z}(k)$ and $[HP(k/k-1)H^T + r]$ are already available in the Kalman filter. The net result is that the great practical difficulties of inverting and evaluating the determinant of a large matrix have been avoided by an approach requiring no matrix inversion at all. The structure of the optimum receiver using this approach is shown in Fig. 4-3.

On-line computations can be greatly reduced by considering use of the Likelihood Function above and some details of filter behavior. First of all, since Likelihood Functions for the various alternatives are to be compared, the term $\sum_{k=1}^{N} \ln 2\pi$ will cancel and hence may be discarded. Further, it frequently happens that the quantity $[HP(k/k-1)H^T + r]$ settles within a few time increments to its steady state value. In such cases, all necessary values of filter gain vectors $B(k)$ and $[HP(k/k-1)H^T + r]$ may be computed beforehand and stored with modest memory requirements. This represents quite a saving since the greater part of computation required by the Kalman filter is associated with updating the variance equation and computing gains. On-line computations are thus reduced to updating the state vector $x(k/k)$ with new observations and adding in the new terms to the Likelihood Function evaluation. From the filter equations listed in Chapter II, this may be seen to amount to approximately $2n + 2$ multiply-add combinations per received sample for each classification alternative, where n is the order of the model.

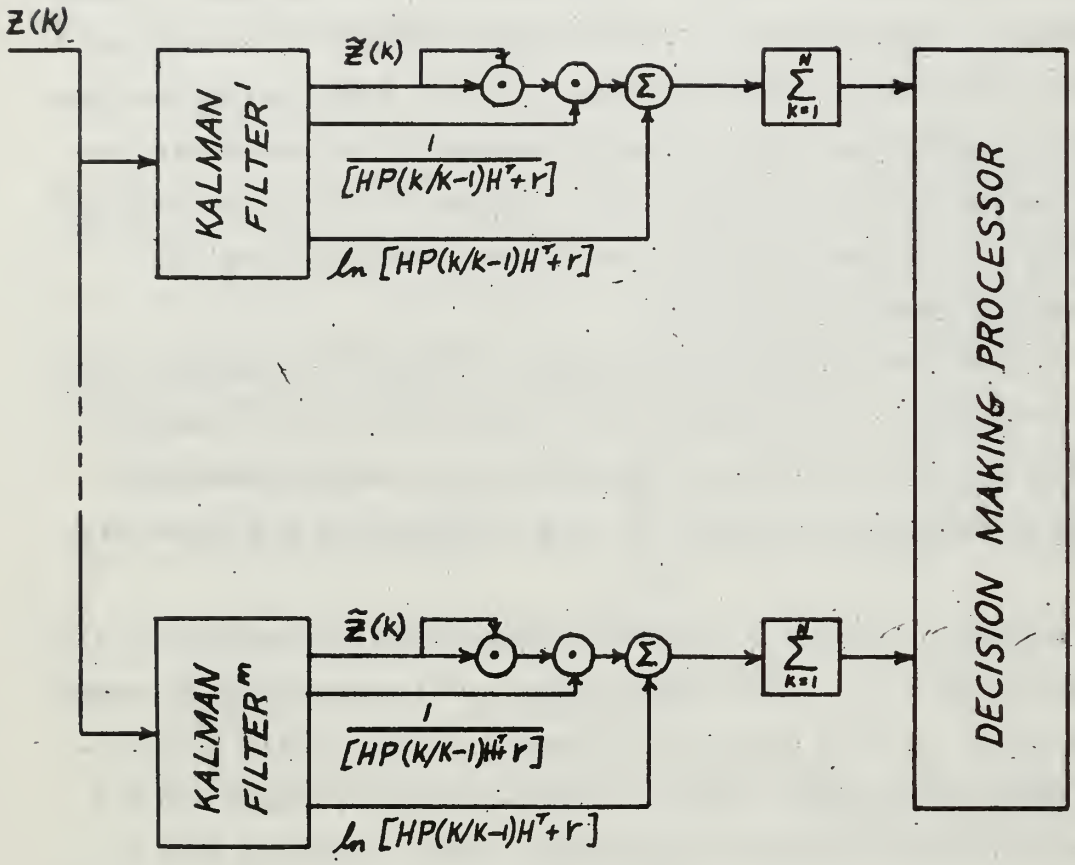


Fig. 4-3 Optimum Receiver Using Recursive Likelihood Function Evaluation

CHAPTER V

AN EXPERIMENTAL STUDY IN RANDOM SIGNAL DETECTION

Finally, utility of theoretical developments in engineering applications rests upon ability of the developed techniques to perform successfully in the "real world". Sensitivity of performance to discrepancies in actual and assumed parameters must be low enough to permit satisfactory operation in the face of such discrepancies. In terms of the detection/classification scheme discussed in Chapter IV, employing recursive evaluation of Likelihood Functions, some points to be investigated are:

(i) Sensitivity to variations in the model quality as measured by the ratio of "correlated" to "uncorrelated" components of the received signal.

(ii) Sensitivity of detection performance to discrepancies between actual and assumed values of received signal power.

(iii) Rapidity with which the technique delivers a clearcut classification decision.

In this chapter, the above points are discussed in the context of an example problem using actual signals.

I. DESCRIPTION OF THE PROBLEM

In the following example, the two-alternative or detection problem is studied. Signals used are hydrophone recordings of (a) sea noise plus sounds of a diesel submarine, and (b) sea noise alone. In discussions below, (a) is referred to as "signal" and (b) as "noise". The resulting problem is as follows: given a segment of received signal record, produce a Maximum Likelihood judgement as to whether the received signal is submarine or noise alone.

Signals used here were originally in analog form on magnetic tape, and were digitized for use in computation using programs described in connection with the example in Chapter III. The basic sampling interval was 200 μs. However, as initial analysis disclosed little signal or noise energy above 800 cycles or so, only every third

sample was employed in computations, extending the sampling interval to 600 μs. The total effective number of samples per block was thus reduced to 1333. Figs. 5-1 and 5-2 display plots of 150 samples of signal and noise, respectively. Fig. 5-3 contains a comparative plot of sample autocorrelation functions of the signal and noise.

For study purposes, ten 4000-sample blocks each of signal and noise were digitized. The first five blocks of each group were used for obtaining models. Testing of the detection procedure was then done using the remaining five blocks in each group.

II. MODEL DETERMINATION

For ease in determination, the autoregressive form of model was selected for the signal and noise sources.. Order of the models was chosen to be six, since the signal was known a priori to have three prominent "lines" at 240, 390 and 625 cps. For less or no a priori knowledge about the signal structure, a procedure might be to begin with low order models and successively increase the order until the comparative power spectral density plots indicate satisfactory representation. For the present problem, recursive estimation of model coefficients as in the example of Chapter III was performed. Resulting models were found to be quite consistent from block to block for the signal. Noise models had consistent dominant poles, but somewhat scattered secondary poles. Figure 5-4 shows the z-plane pole locations for signal models from the first five blocks of signal data. Figure 5-5 shows the same information for noise models.

Selection of a specific model for use in detection might be made by, say, averaging the corresponding coefficients of the five sample-derived models for each case. A simpler but less precise method would be to just select a representative model from each group. The latter method was used for the experiments here. Figures 5-6 and 5-9 contain z-plane pole plots for the selected signal and noise models respectively. Figures 5-7 and 5-8 display comparative plots of autocorrelation functions and power spectral densities derived from the

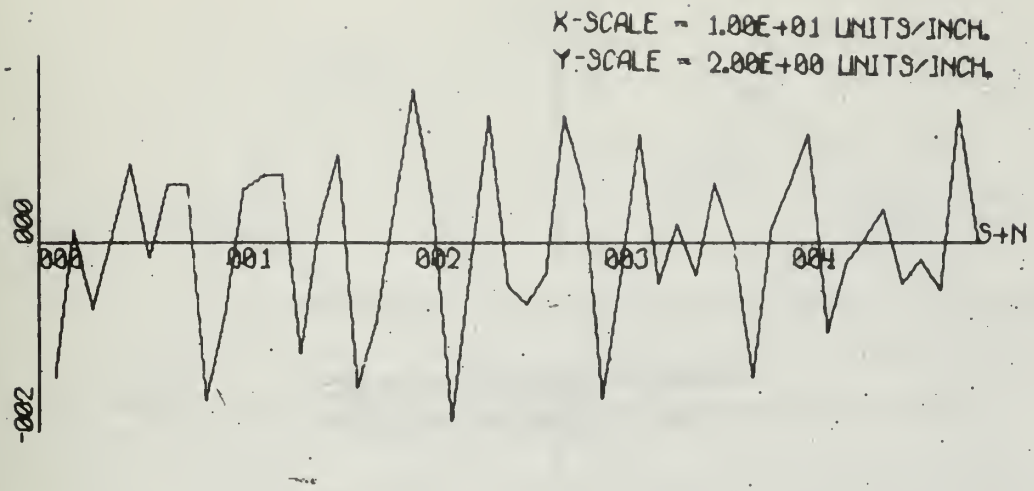


Fig. 5-1 Sea Noise Plus Diesel Submarine ("Signal")

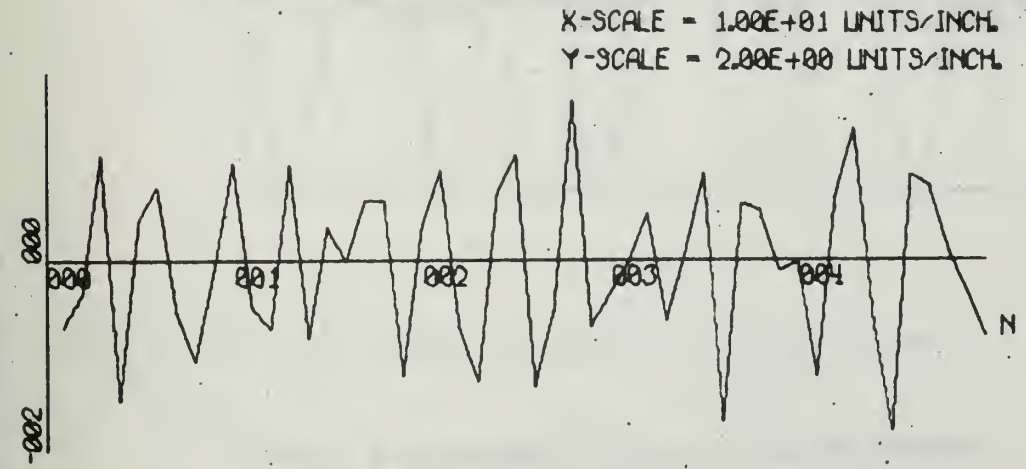


Fig. 5-2 Sea Noise ("Noise")

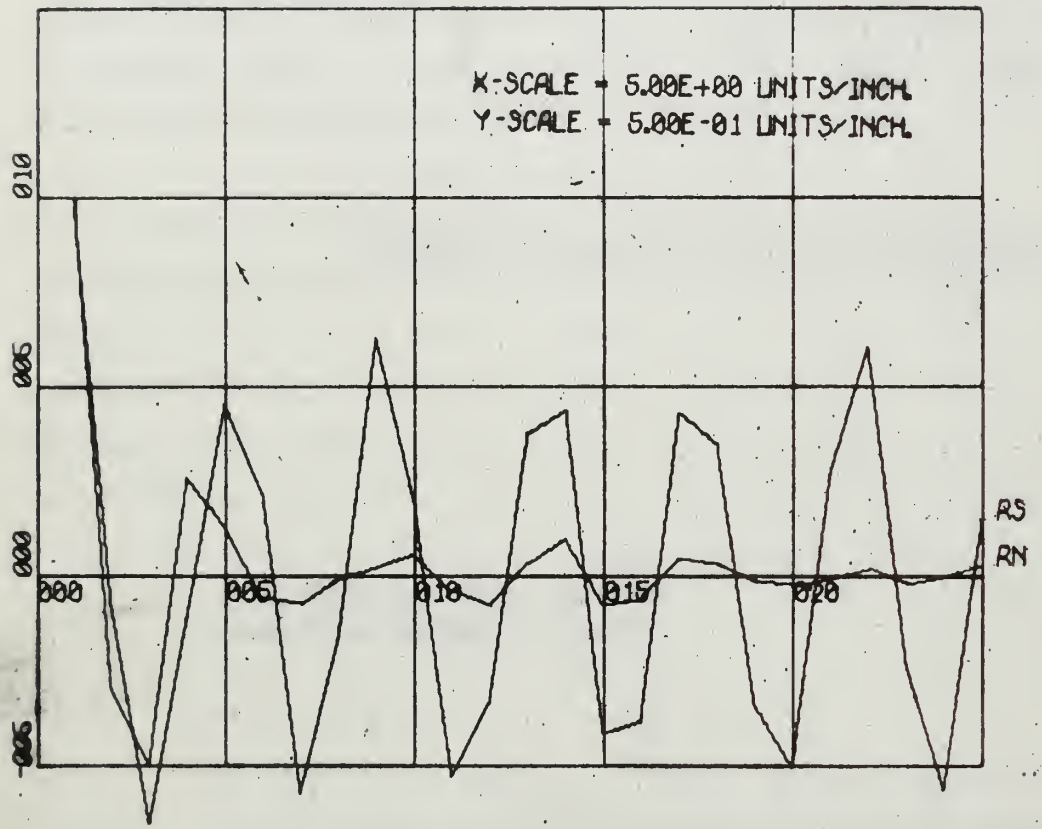
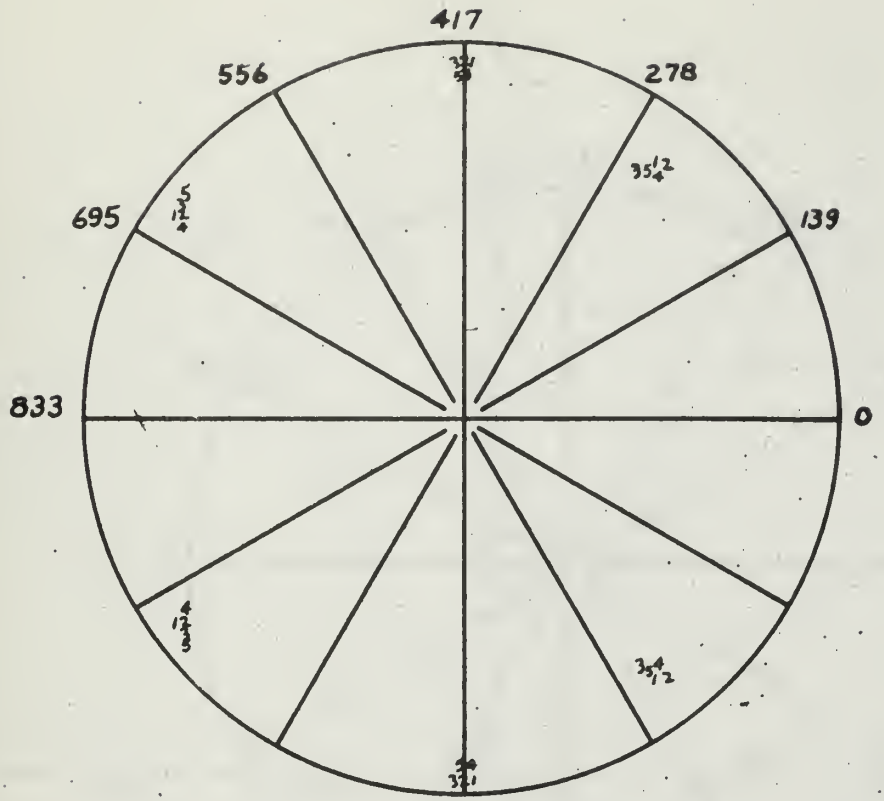
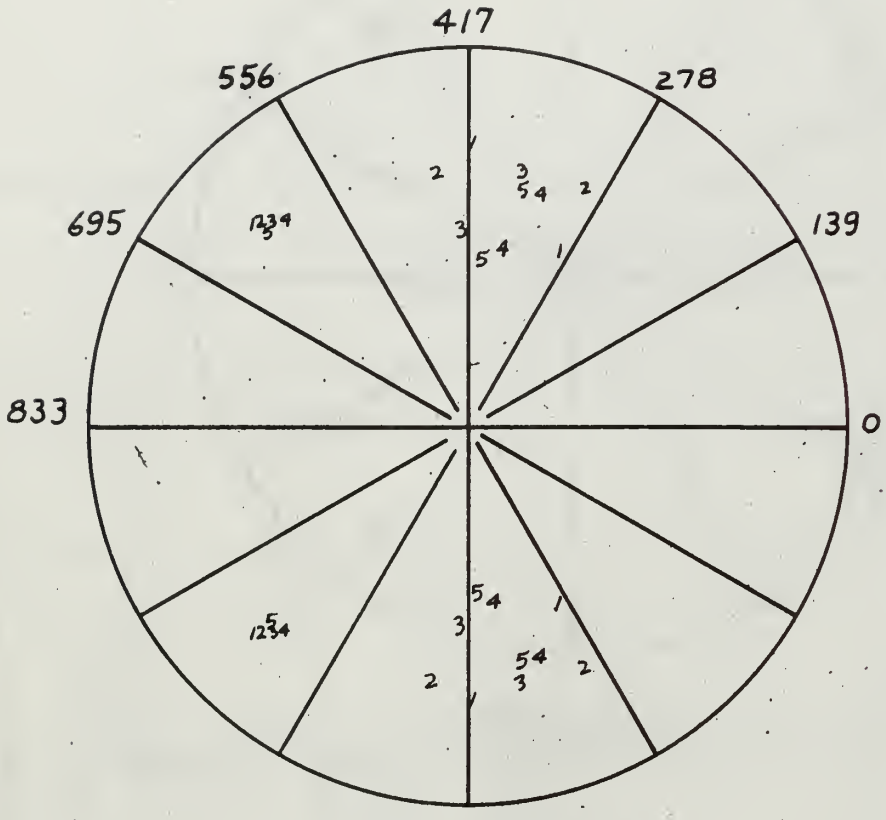


Fig. 5-3 Autocorrelation Function of "Signal" and "Noise"



PLOT	BLOCK
1	1
2	2
3	3
4	4
5	5

Fig. 5-4 Z-Plane Poles of Autoregressive Signal Models



PLOT	BLOCK
1	13
2	14
3	15
4	16
5	17

Fig. 5-5 Z-Plane Poles of Autoregressive Noise Models

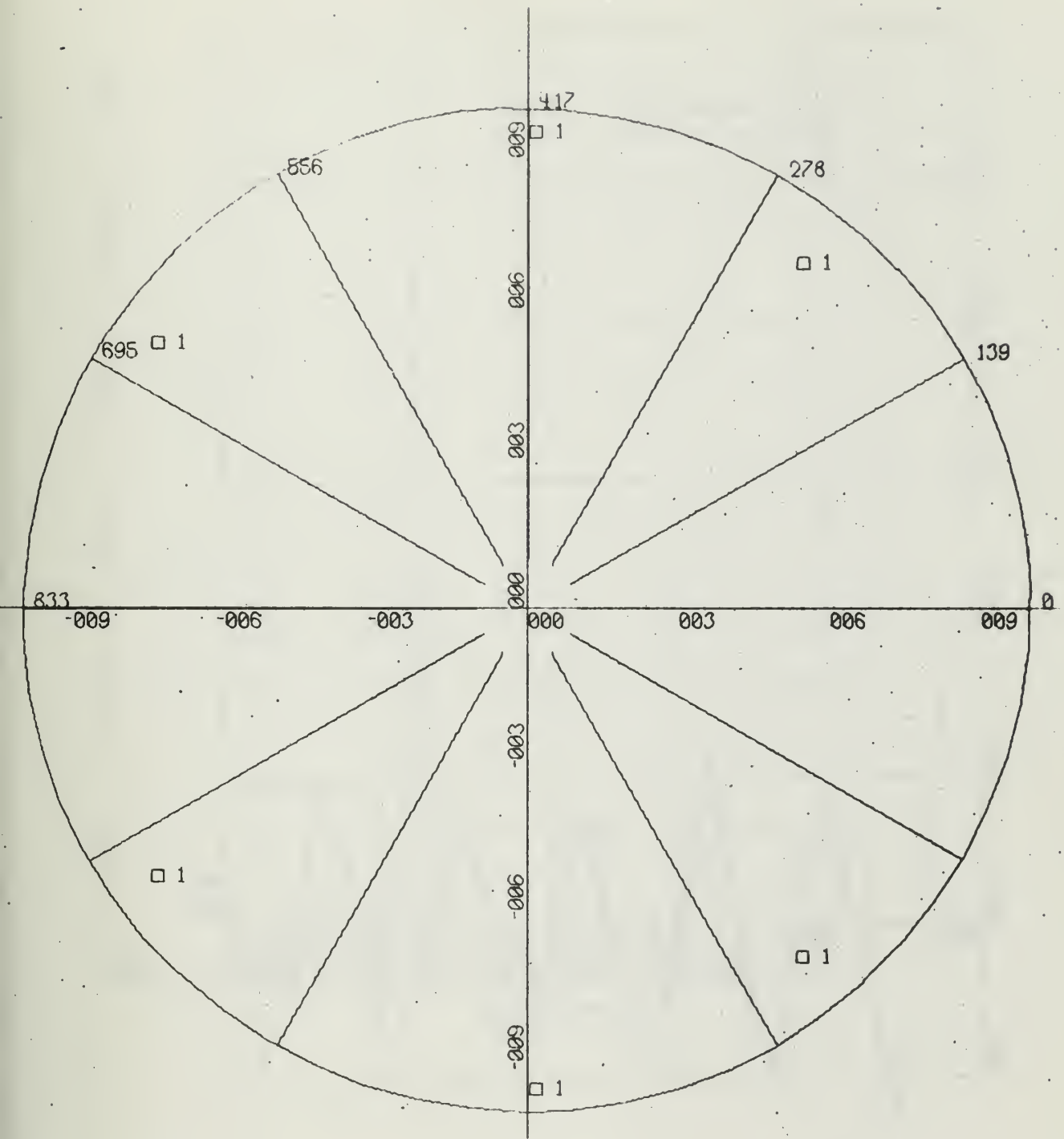


Fig. 5-6 Poles of Representative Signal Model

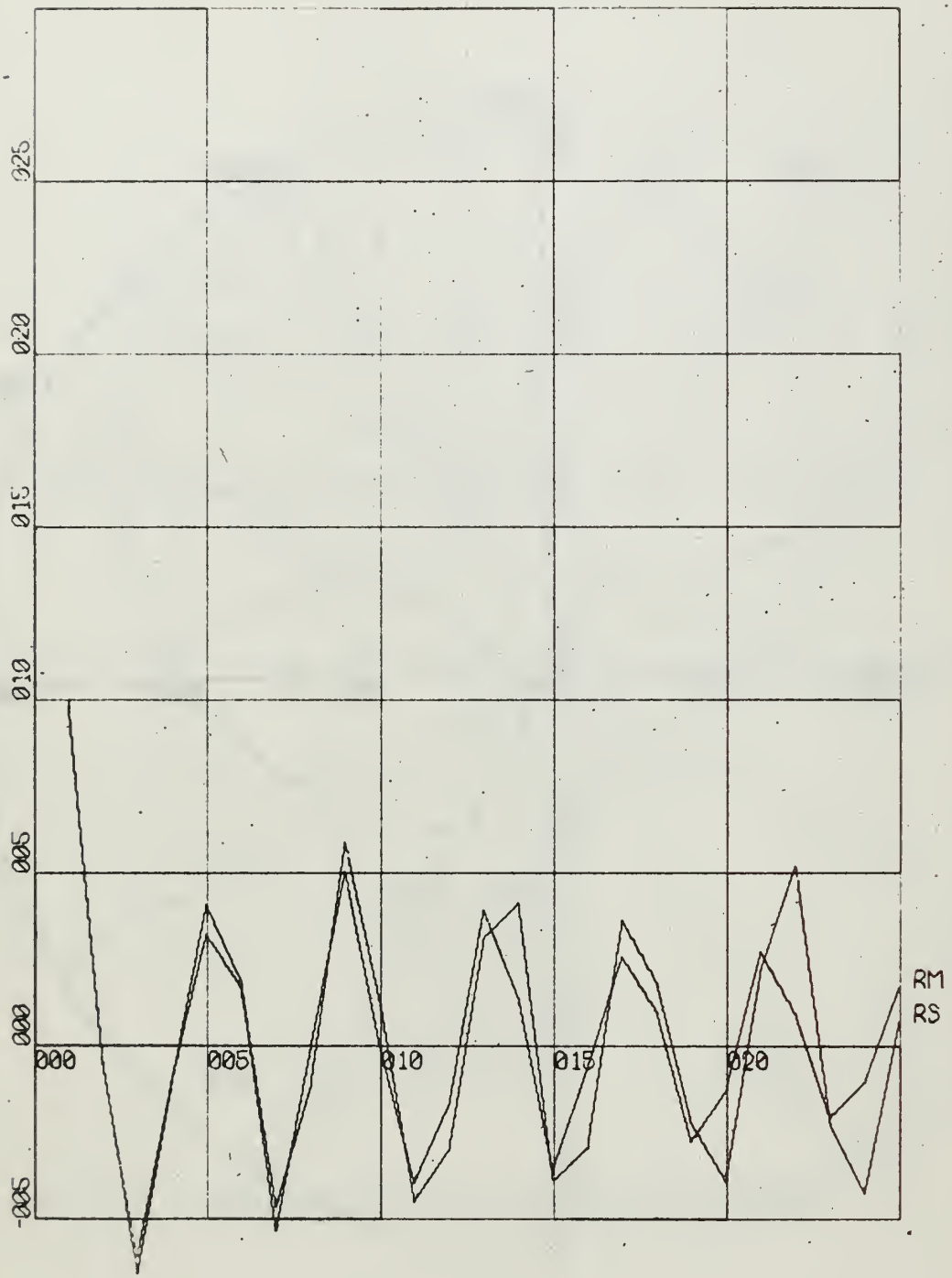


Fig. 5-7 Autocorrelation Functions from Signal Data and Signal Model

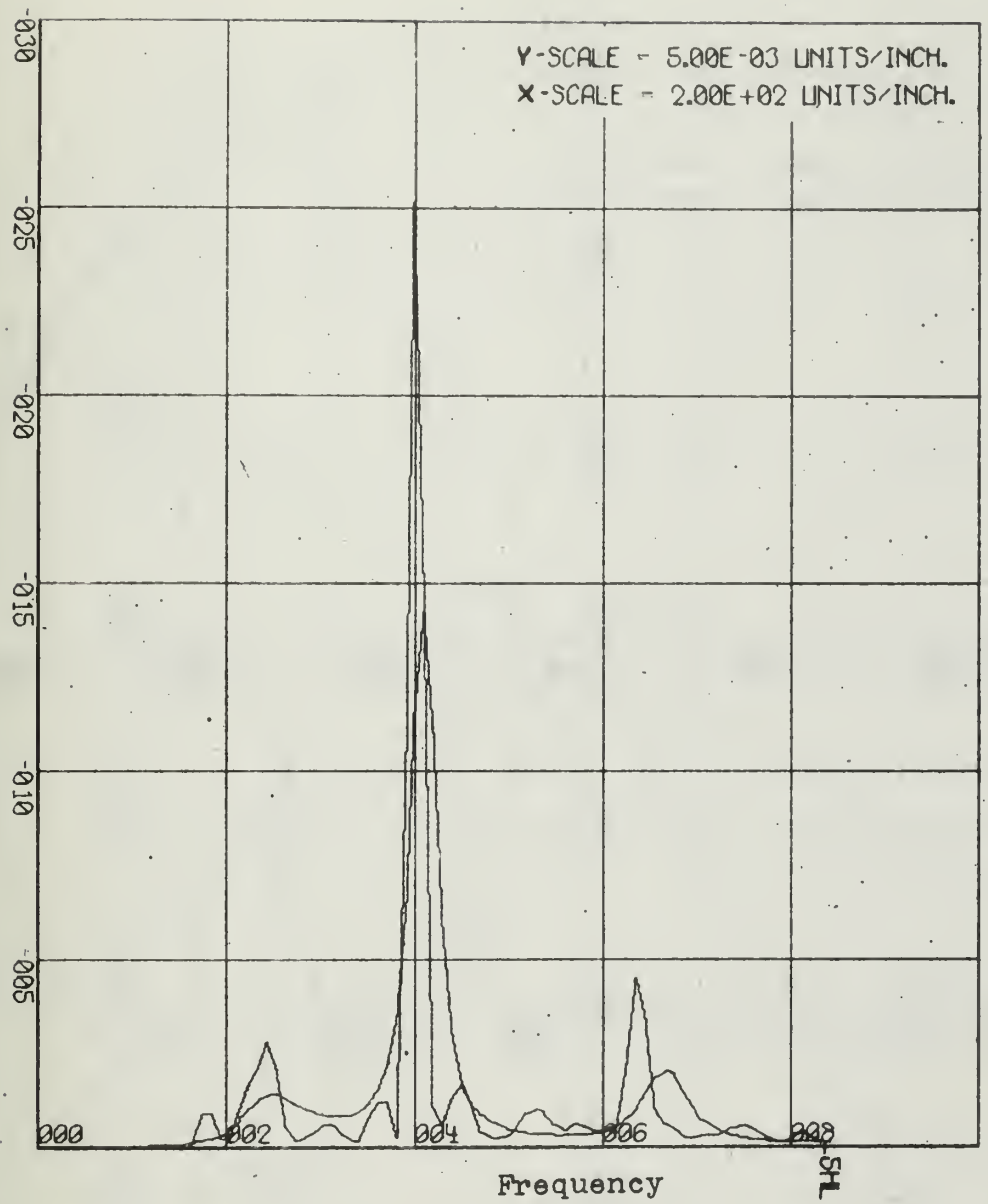


Fig. 5-8 Power Spectral Densities from Signal Data and Signal Model

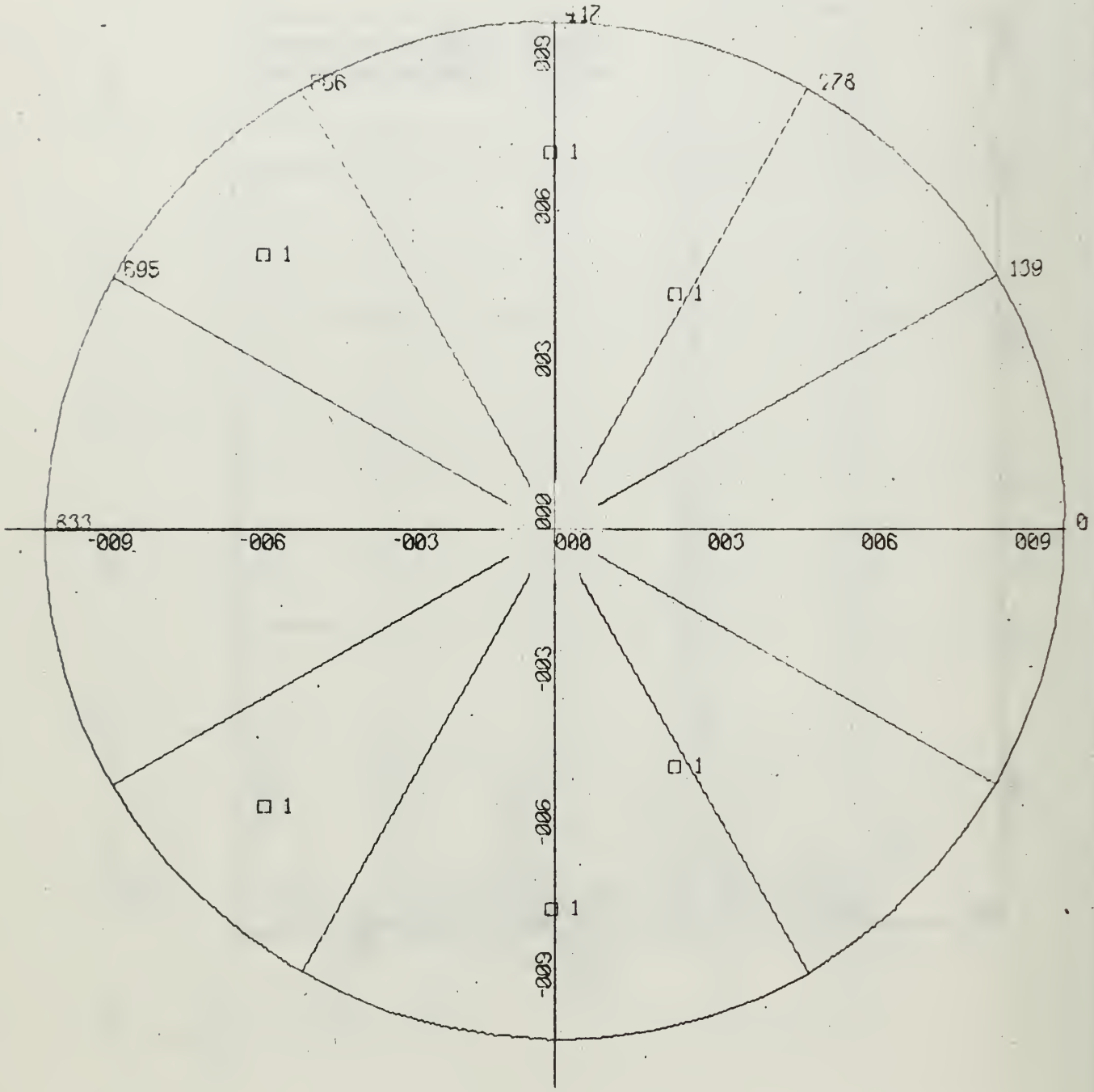


Fig. 5-9 Poles of Representative Noise Model

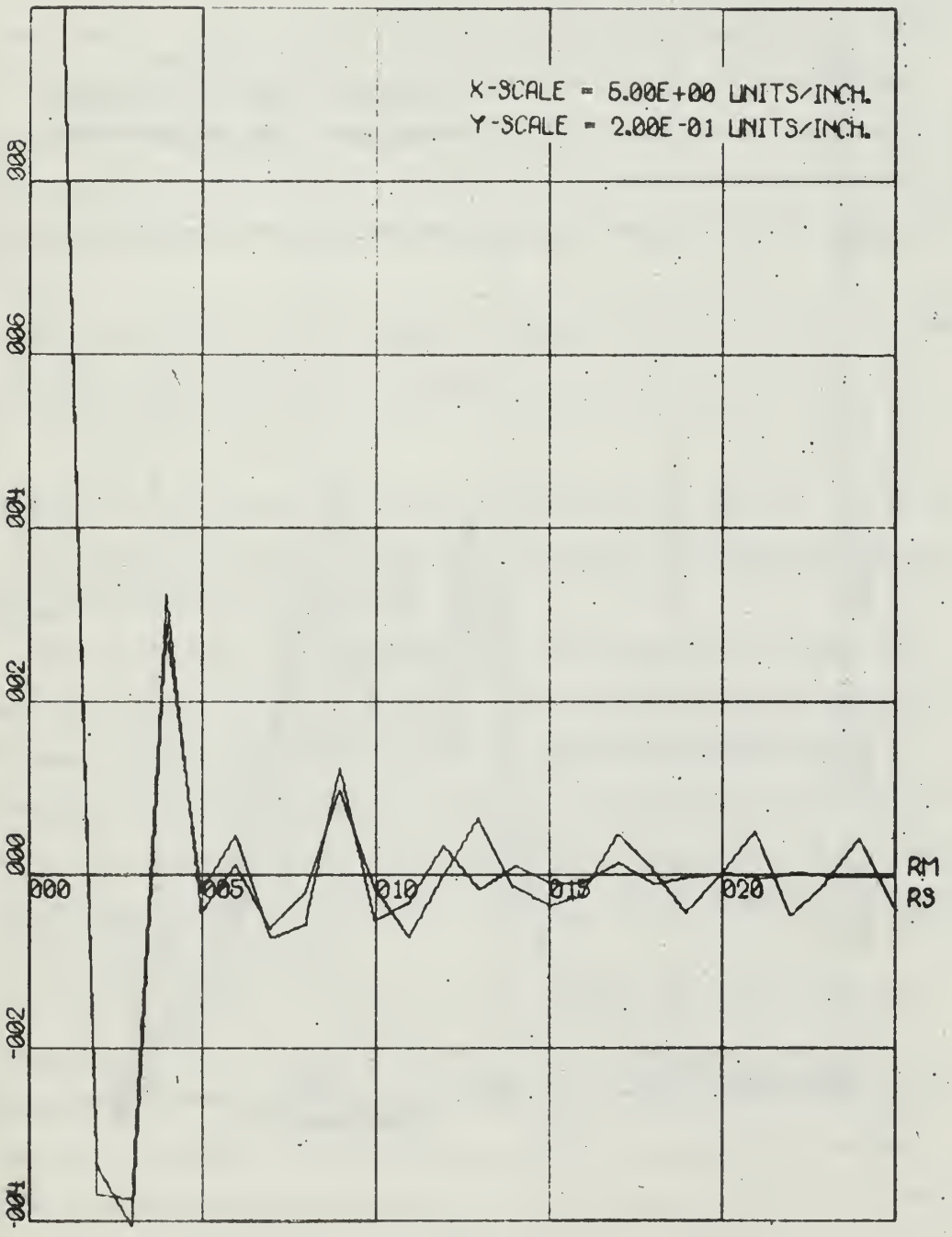


Fig. 5-10 Autocorrelation Functions from Noise Data and Noise Model

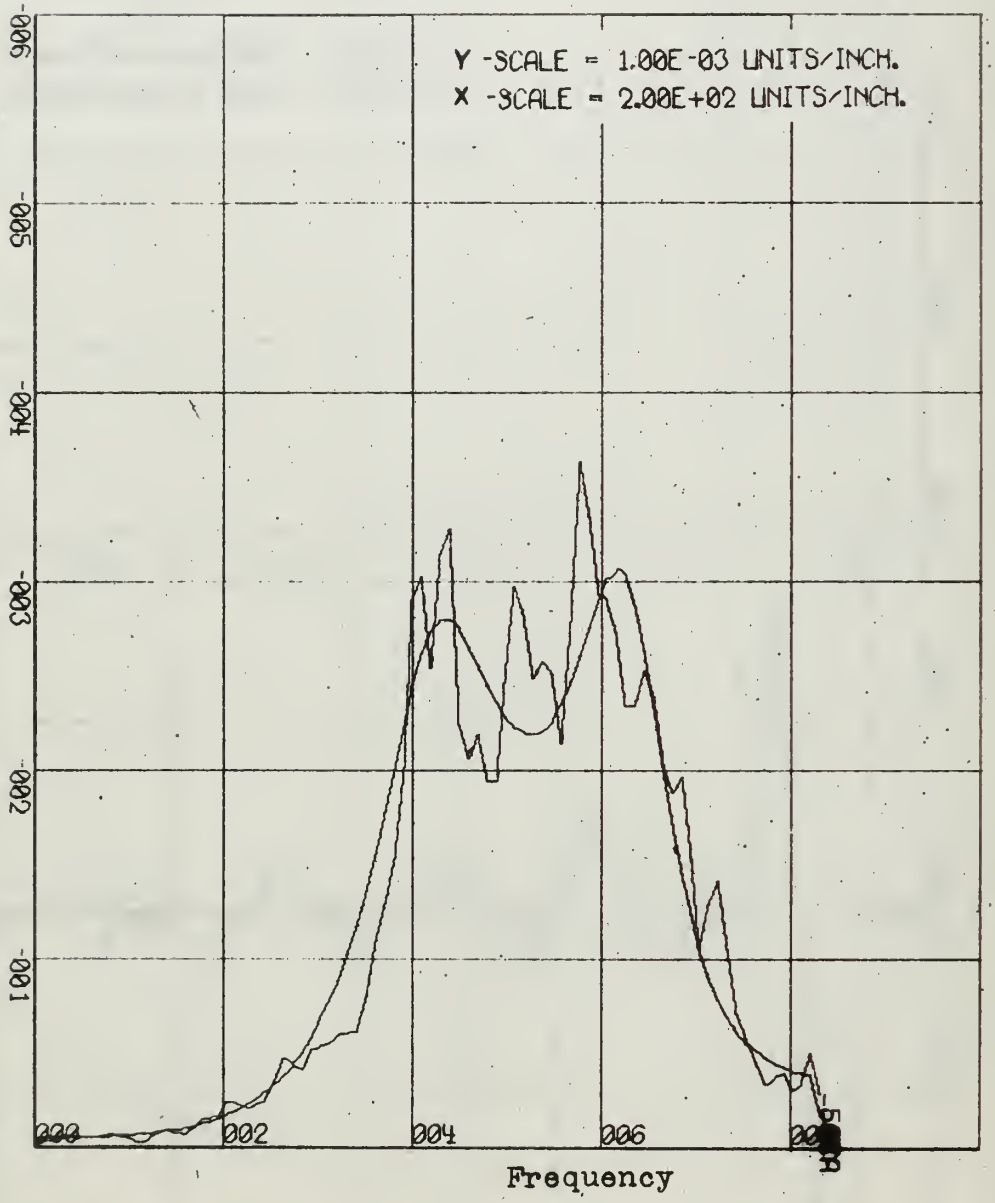


Fig. 5-11 Power Spectral Densities from Noise Data and Noise Model

data sample and model respectively. Figures 5-10 and 5-11 convey the same information about the noise model.

III. STUDY OF PERFORMANCE

For an honest evaluation of discrimination power of the technique being tested, it is important that signal classification be made solely on the basis of differences in correlation structure of the alternate possibilities and not on energy levels. For this reason, each batch of 4000 samples from which sub-blocks of 100 samples were processed by the detector was normalized to have a mean squared sample value of unity.

In terms of the discrete signal process model pictured in Fig. 2-2, this means that $E[z^2] = 1$. But since

$$E[z^2] = E[y^2] + r,$$

it remains to be determined what is the most likely ratio of $E[y^2]$ to r. The ratio is a measure of the degree to which the model obtained represents the actual observed signal.

Using a value of .9 for this ratio, the Likelihood Function for each alternative is computed as a function of the number of samples processed and is plotted in Fig. 5-12. Since the quantity -2 ln Likelihood Function is actually computed and displayed, higher likelihoods are represented by lower ordinates on the graph. In Fig. 5-12 samples processed were actually "signal". Hence correct classification is indicated by the graph.

Program QTEST then evaluates the Likelihood Function for each classification alternative for values of this ratio from .1 to .9. Figures 5-13 to 5-16 display the resulting values of the Likelihood Function versus the number of data samples processed. Note that correct classification was achieved for all values of the ratio between .1 and .9. It might also be noted that the ratio value of .9 gave the greatest margin of correct classification. Correct classification was also consistently obtained in the same manner for all blocks of independent data (not utilized in model determination).

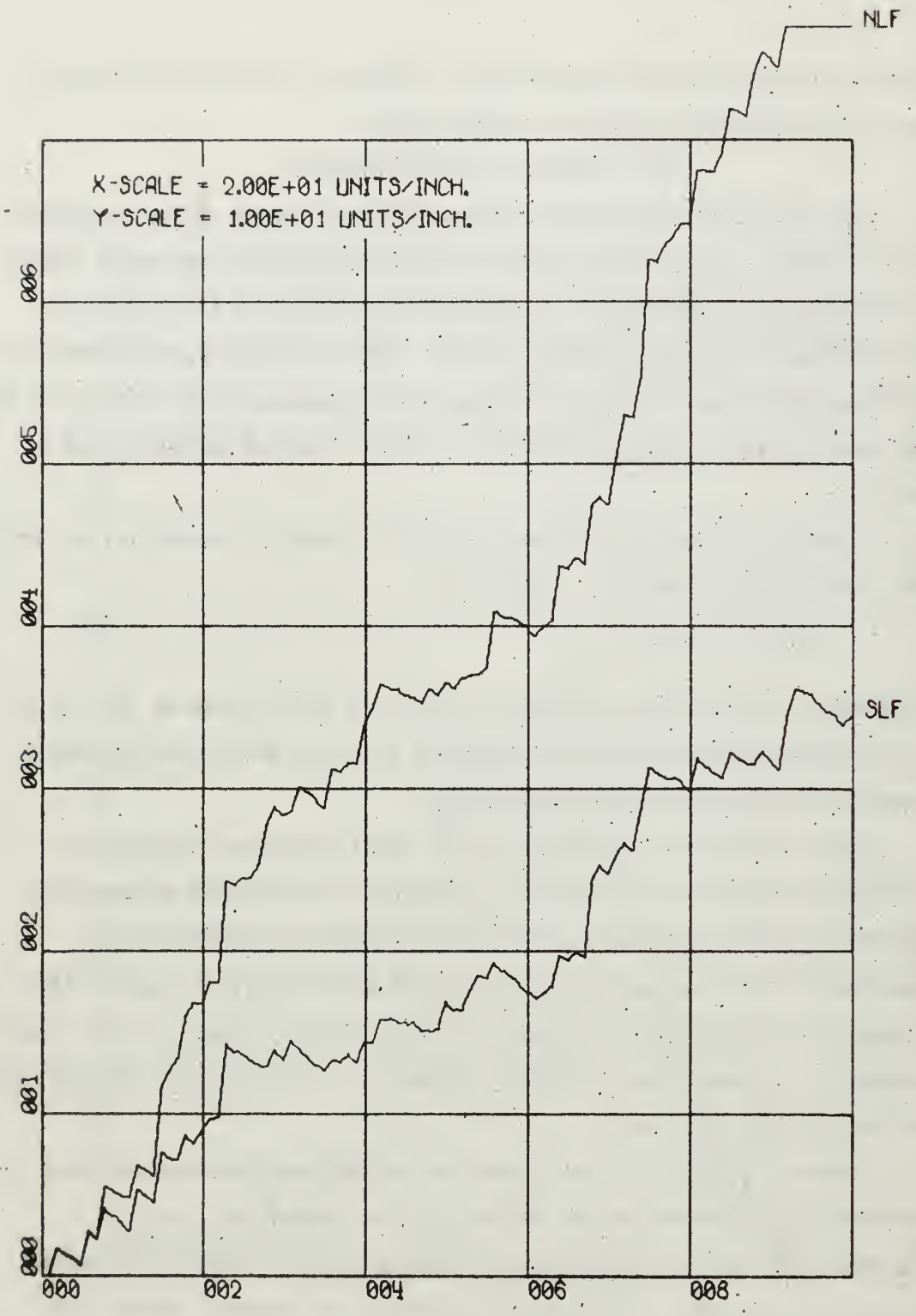


Fig. 5-12 Likelihood Functions for Signal Data Computed Using Signal and Noise Models

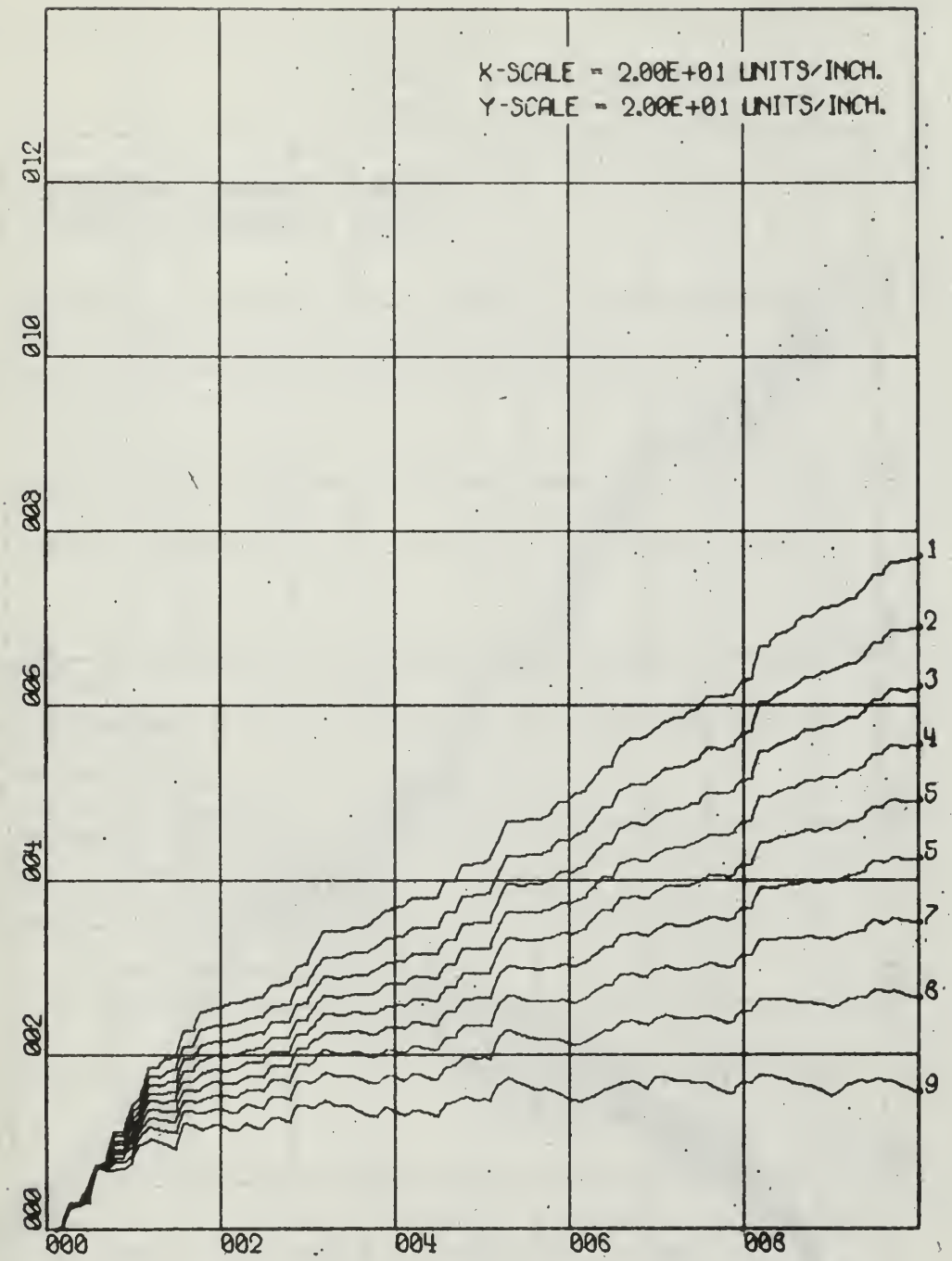


Fig. 5-13 Likelihood Functions of Signal Using Signal Model

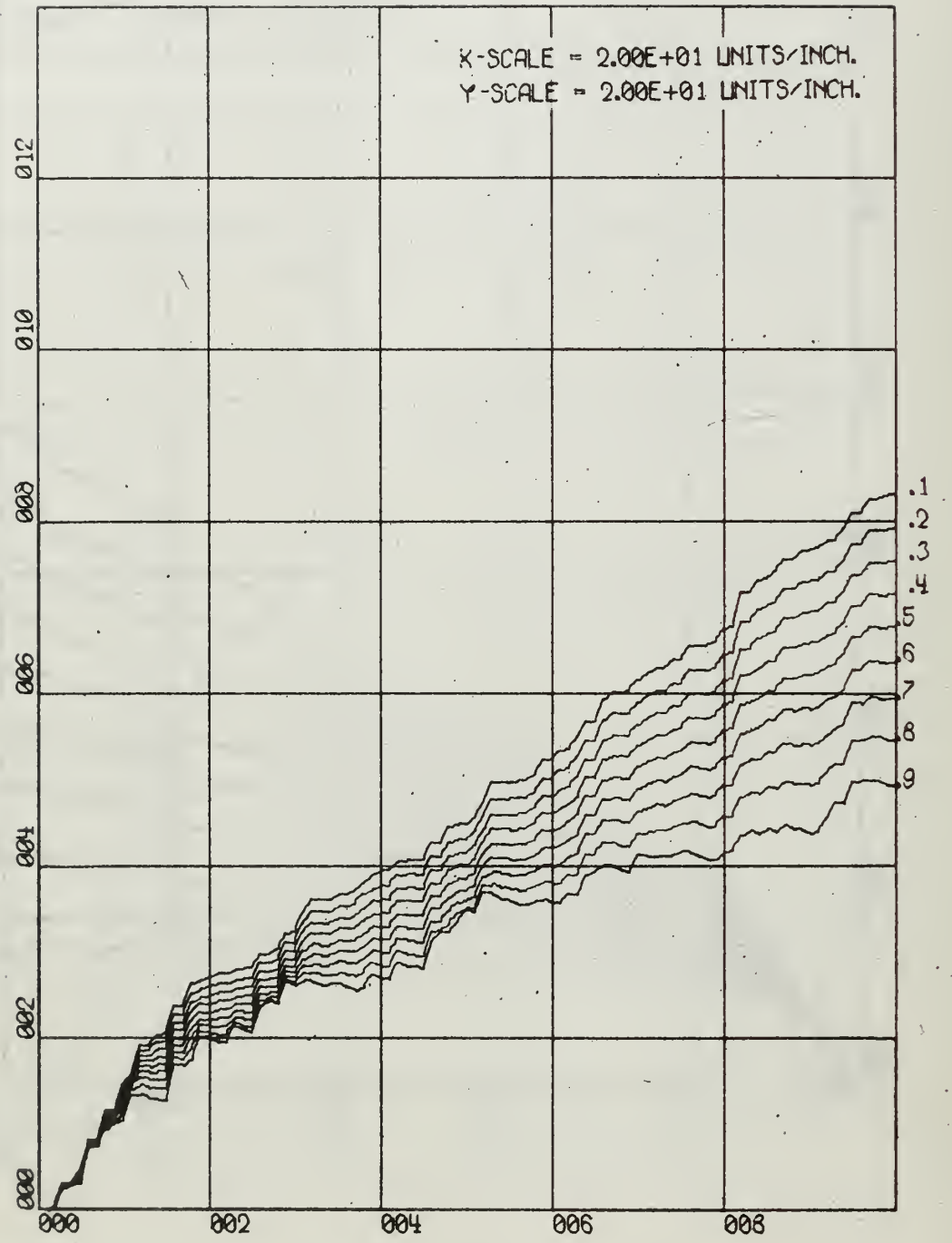


Fig. 5-14 Likelihood Functions of Signal Using Noise Model

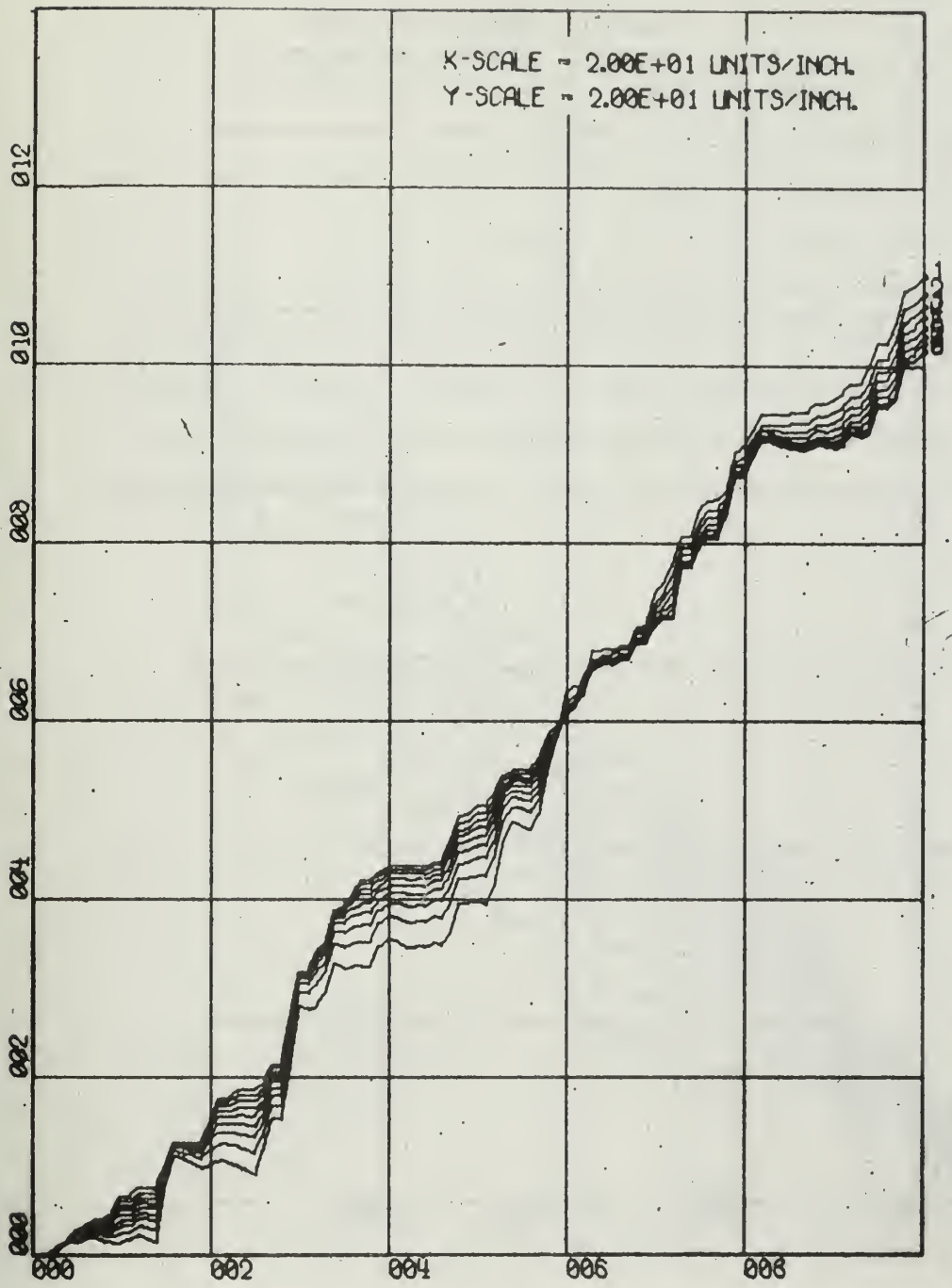


Fig. 5-15 Likelihood Functions of Noise Using Signal Model

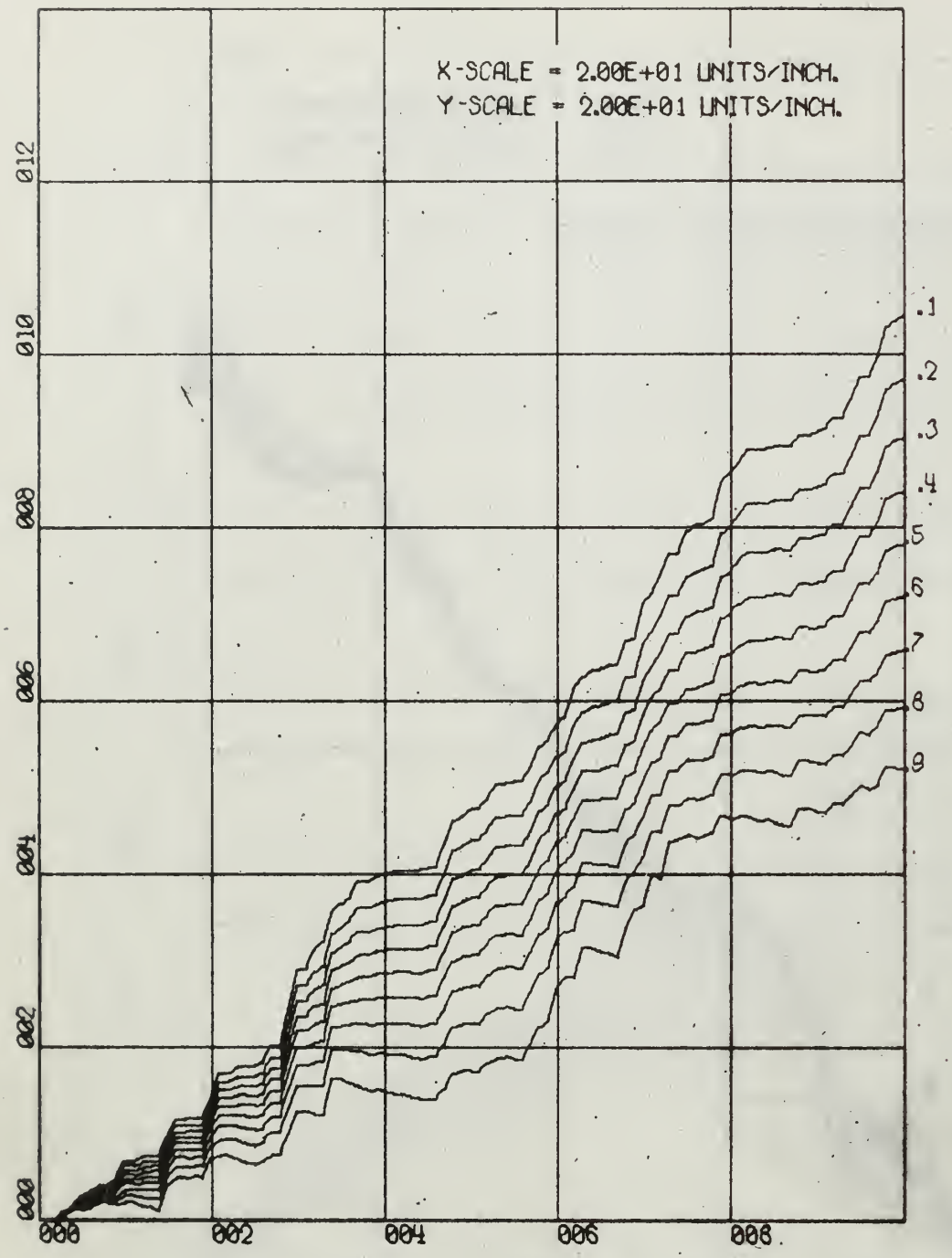


Fig. 5-16 Likelihood Functions of Noise Using Noise Model

The normalization process described above is roughly equivalent, in practical terms, to automatic gain control (AGC). It is important that the detection scheme continue to function efficiently under deviations between actual and assumed amplitudes (and corresponding mean square values). Hence a test was undertaken to check classification efficiency against signals with actual mean square values ranging from one fourth to four times the assumed value. Again, consistent correct classification was maintained. Figures 5-17 to 5-24 contain the resulting Likelihood Function plots for actual mean square values equal to one fourth and four times the assumed value, in all cases equal to unity.

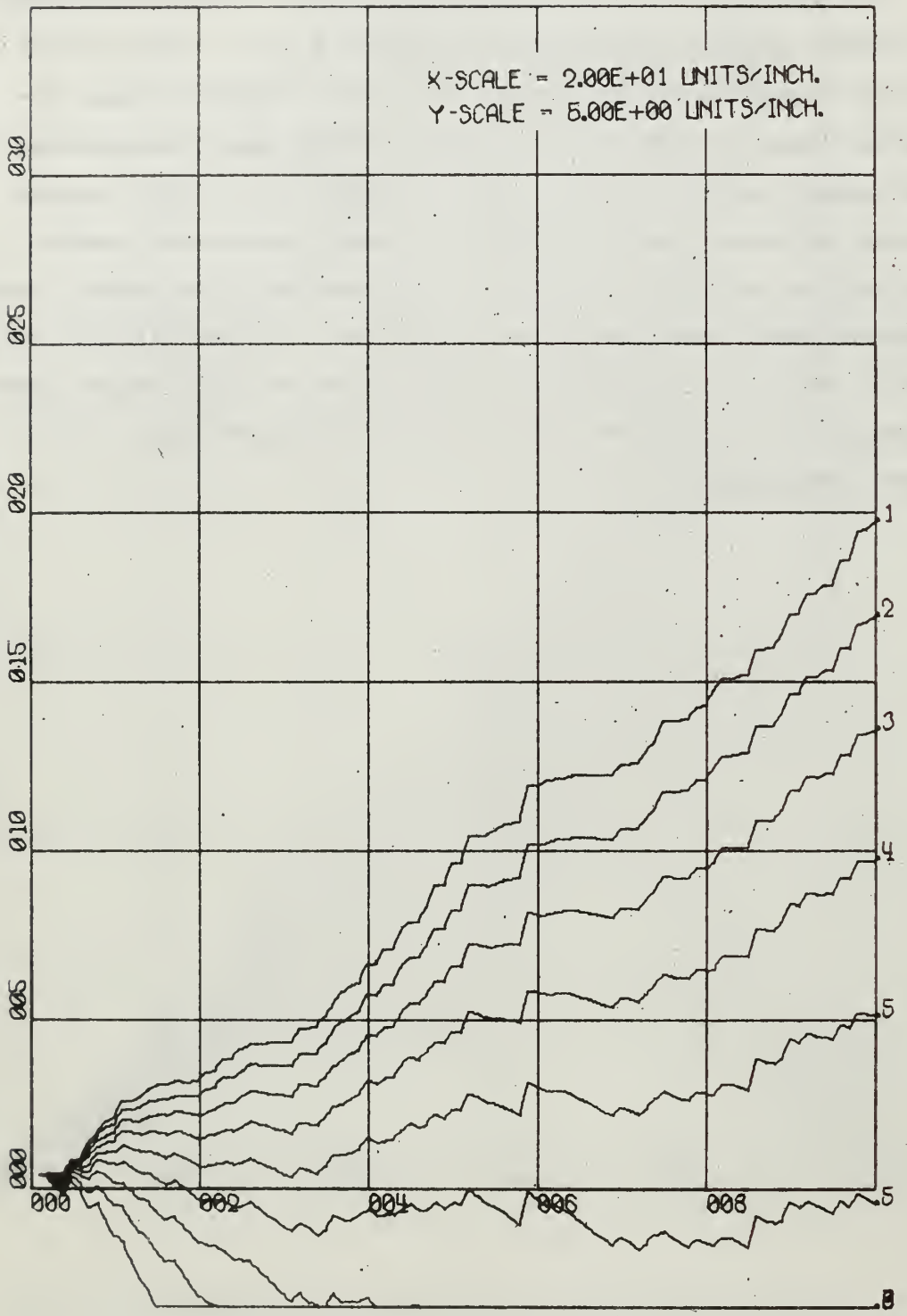


Fig. 5-17 Likelihood Functions of Signal Using Signal Model - Mean Square Signal Received Equal One-fourth Assumed Value

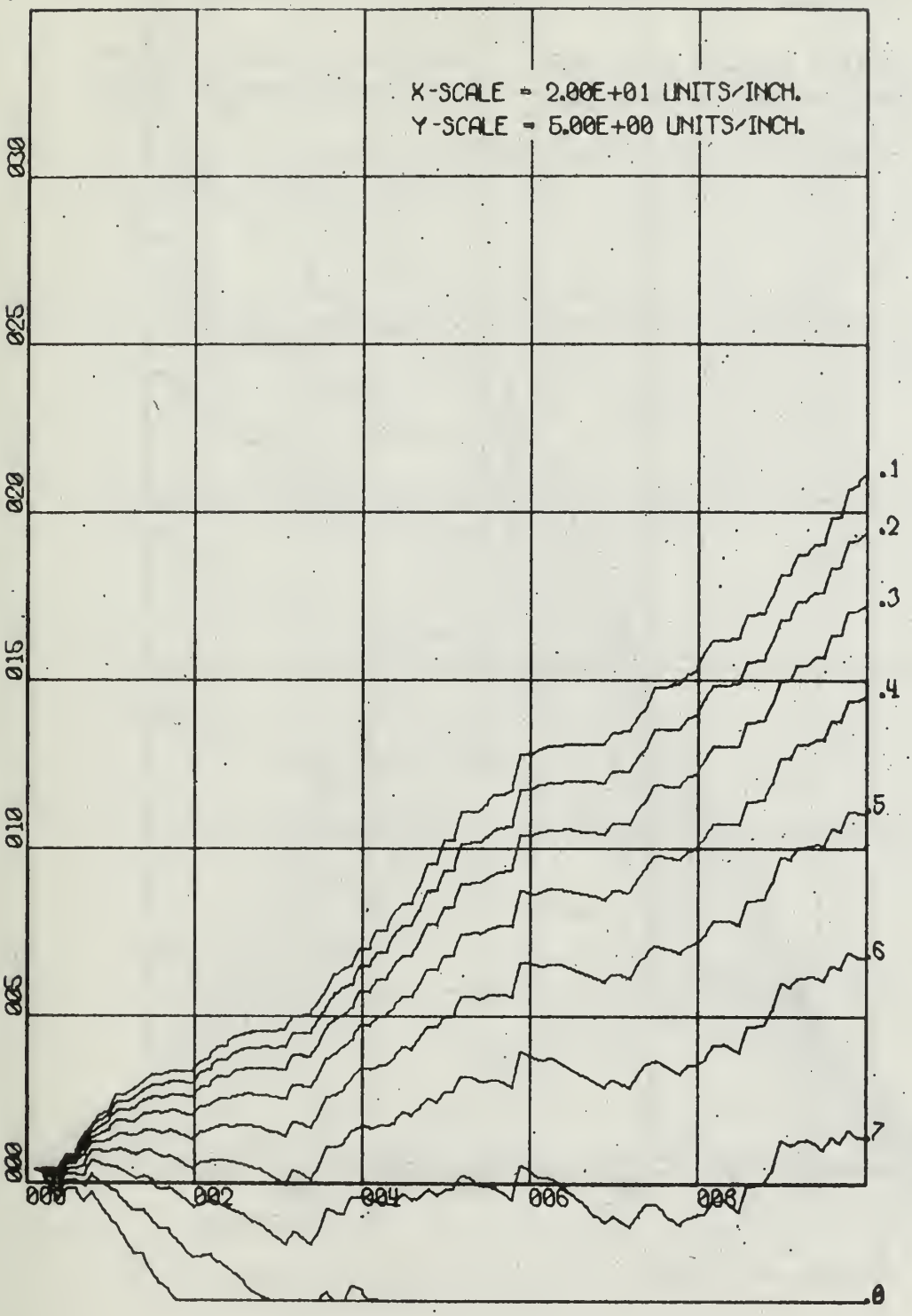


Fig. 5-18 Likelihood Function of Signal Using Noise Model - Mean Square Signal Received Equal One-fourth Assumed Value

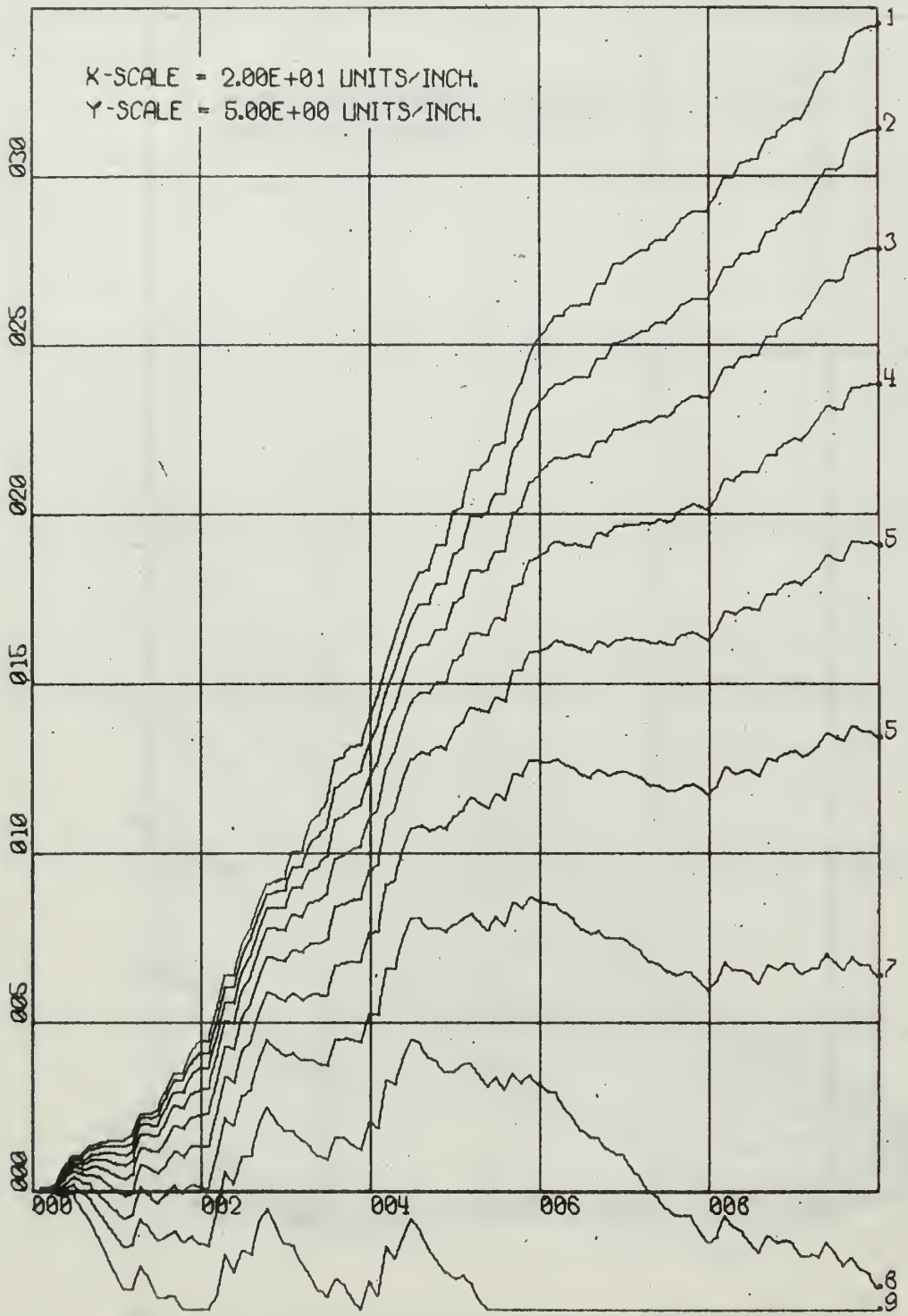


Fig. 5-19 Likelihood Function of Noise Using Signal Model - Mean Square Signal Received Equal One-fourth Assumed Value

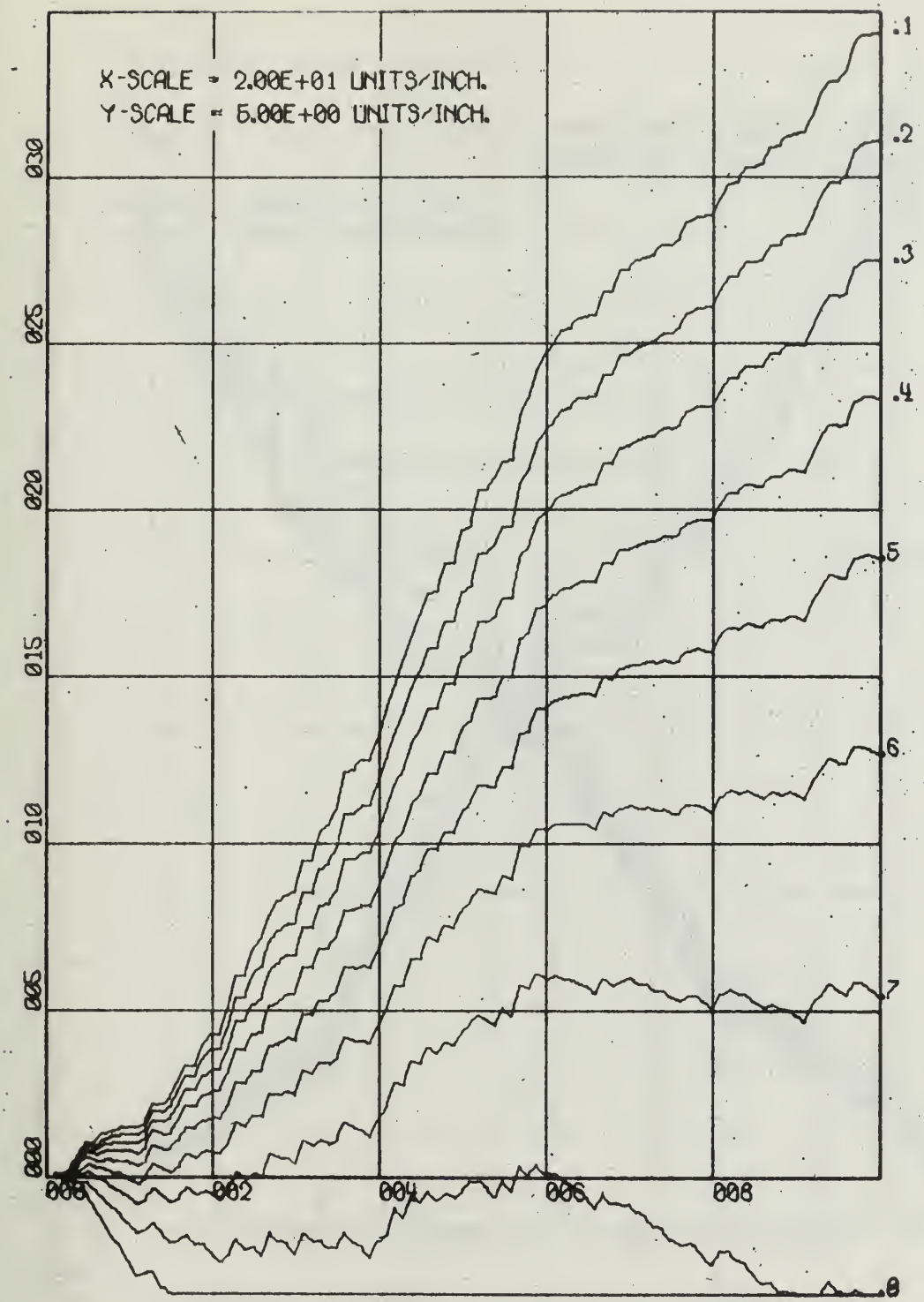


Fig. 5-20 Likelihood Function of Noise Using Noise Model - Mean Square Signal Received Equal One-fourth Assumed Value

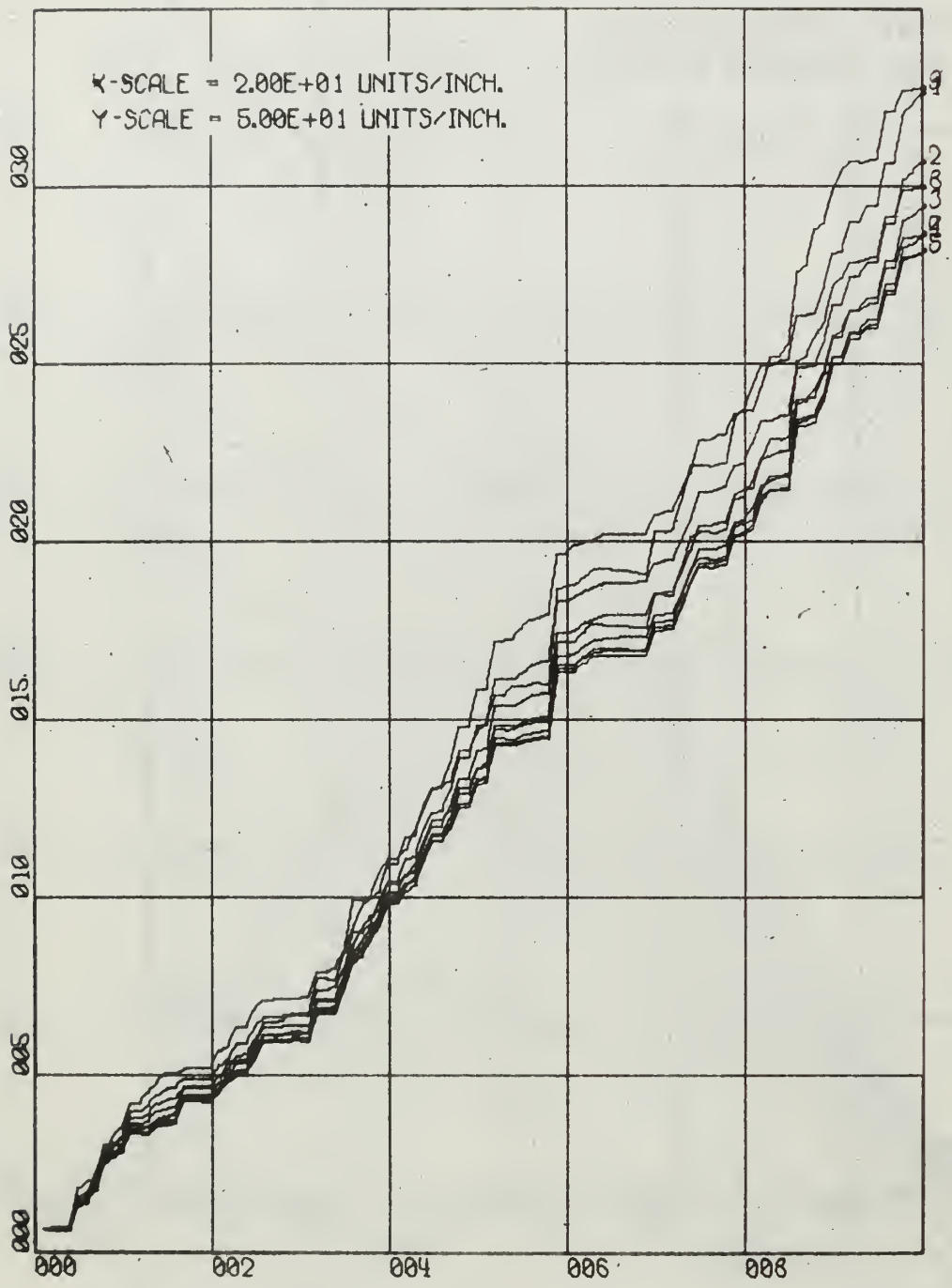


Fig. 5-21 Likelihood Function of Signal Using Signal Model - Mean Square Signal Received Equal Four Times Assumed Value

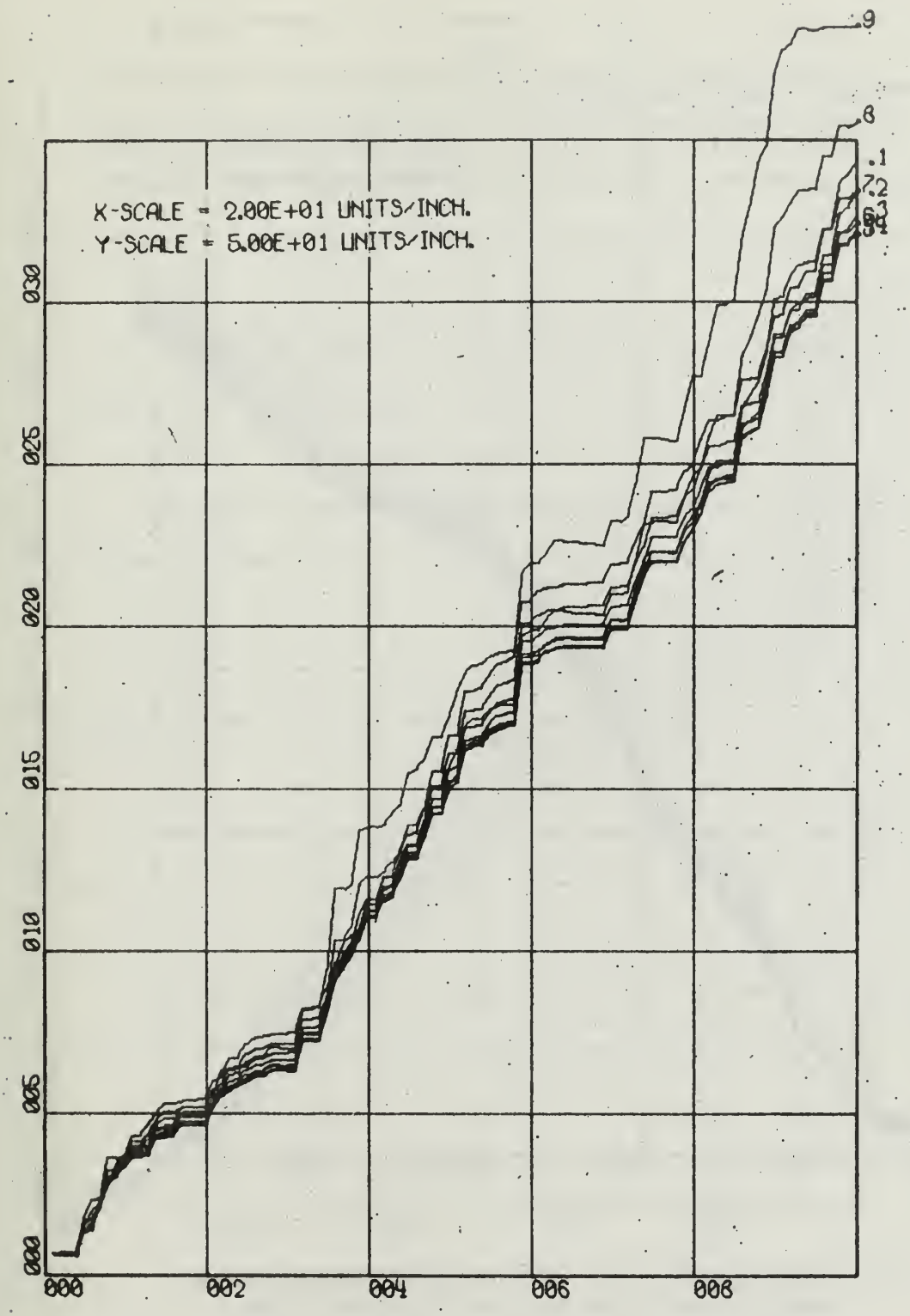


Fig. 5-22 Likelihood Function of Signal Using Noise Model - Mean Square Signal Received Equal Four Times Assumed Value

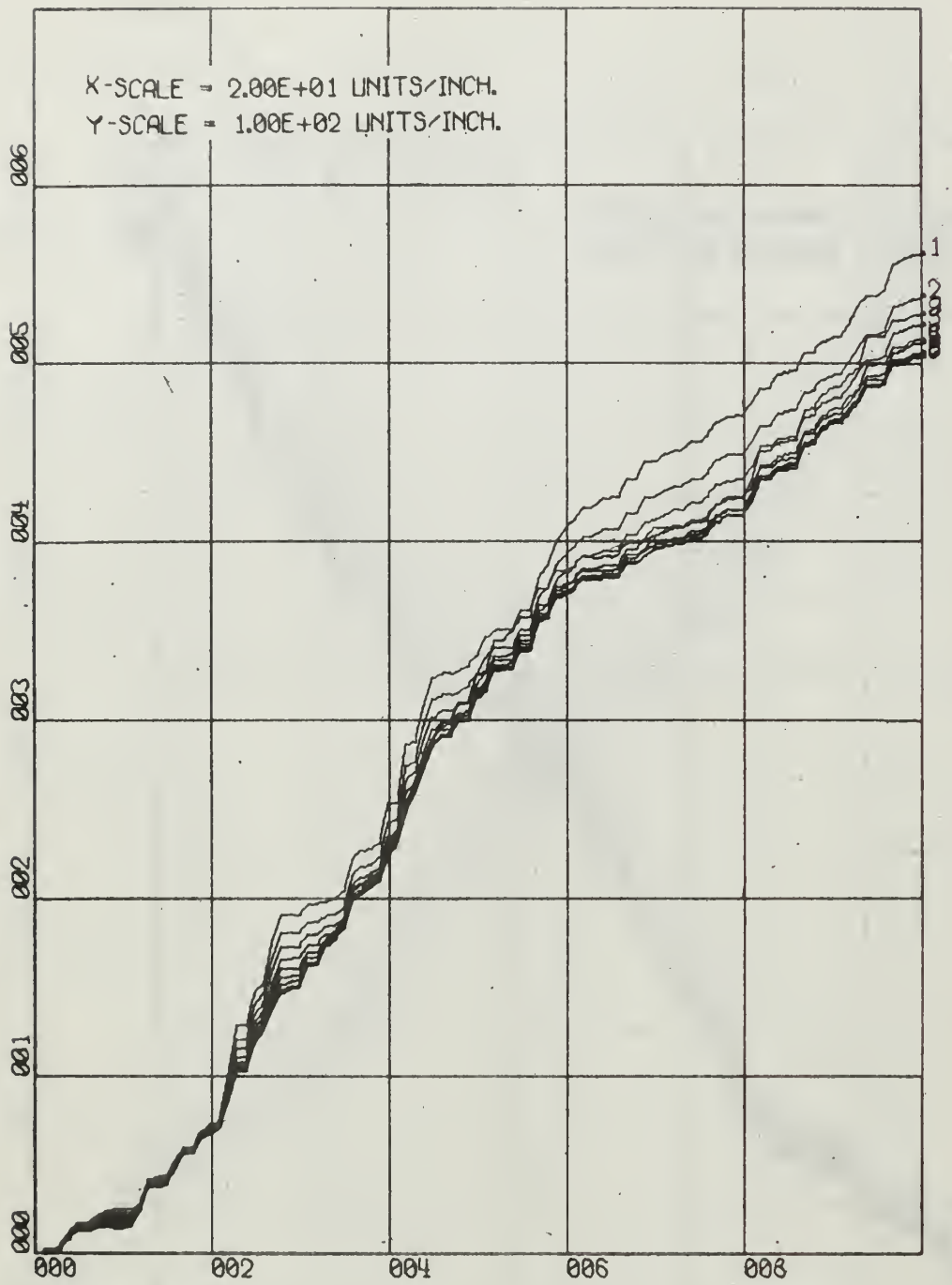


Fig. 5-23 Likelihood Function of Noise Using Signal Model - Mean Square Signal Received Equal Four Times Assumed Value

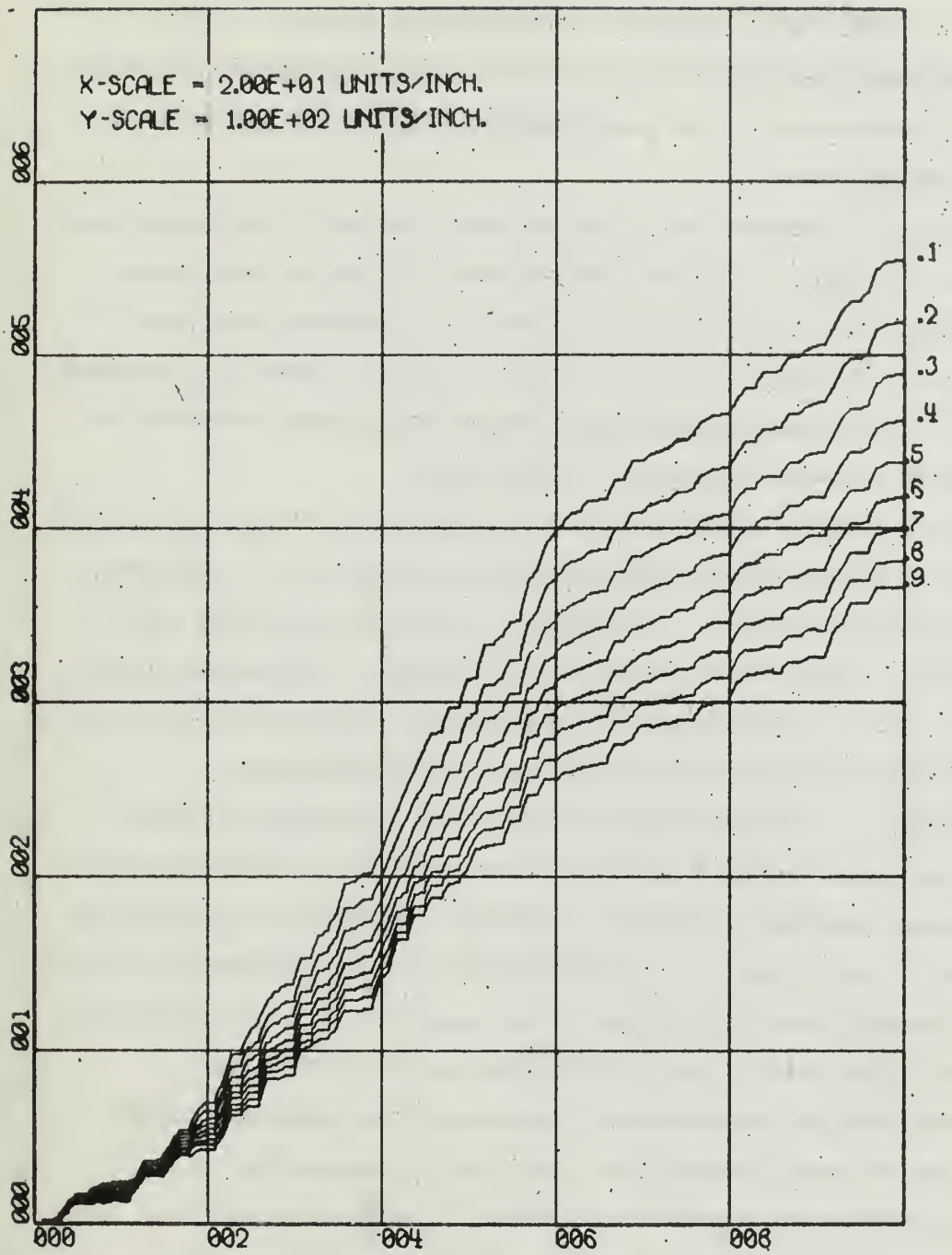


Fig. 5-24 Likelihood Function of Noise Using Noise Model - Mean Square Signal Received Equal Four Times Assumed Value

CHAPTER VI

RECOMMENDATIONS FOR FURTHER WORK

A frequent happening in applied research is that there are as many new questions raised as old questions resolved. Such has been the case in this instance.

Perhaps the problem most ripe for attack is that of estimating numerator coefficients of the linear filter model for random time series. This is equivalent to estimation of the moving average coefficients in a mixed auto-regressive-moving average model. A promising approach might lie in numerical processing of the residuals after recursive determination of the autoregressive coefficients.

Further study of applications of the methods used here to practical problems in signal detection/classification would also be interesting and potentially valuable. In particular, multiple-alternative signal classification should be interesting to consider. Performance in such problems should be improved by better models resulting from successful determination of linear filter model numerator coefficients.

Reliability of detection/classification as a function of record length processed and as a function of some measure of difference between power spectral densities of alternate classifications should be valuable to investigate. For the detection problem, differences between power spectral densities of signal plus noise and noise alone relate to signal-to-noise ratio in the conventional problem formulation.

Classification methods used herein also hold potential contributions in character recognition. Required for application of the methods here are temporally vice spatially distributed patterns. However, the scanning process used in several present approaches to character recognition already produces the necessary spatial-temporal transformation. The advantage to be gained should be an insensitivity to character translation, a problem which plagues many present approaches.

Finally, the problem of model determination for vector-valued random time series remains. Some work on the problem has been done,

and is described in [3], [30]. A comparative study of work referenced and the general linear filter model should hold some promise of benefiting the general analysis of vector-valued random time series both in engineering and other disciplines.

APPENDIX I

OPTIMAL ESTIMATION OF AN ARBITRARY LINEAR COMBINATION OF SIGNAL GENERATING PROCESS STATE VARIABLES

Assume the usual model of the signal generating process with state variables $x(k)$ and noisy observations $z(k) = Hx(k) + u(k)$. It is desired to estimate an arbitrary linear combination of the state variables $s = Dx$ based on the noisy observations z. The optimal linear estimate (in the Bayes sense, with a quadratic loss function) can be shown [17], [35] to be the expected value of the conditional distribution

$$p[s(k)/z(k), z(k-1), \ldots . z(1)]$$

Since the process is linear, and since the excitation and noise sources are gaussian, the conditional distributions of both x and s given measurements z are gaussian. The problem at hand will be solved if the conditional distribution of s given z can be obtained, from which the mean may be extracted.

Consider first the conditional distribution of x given the measurements z. By Bayes rule,

$$p[x(k)/z(k), z(k-1), \ldots z(1)]$$
$$= \frac{p[z(k)/x(k), z(k-1), \ldots z(1)]\, p[x(k)/z(k-1), \ldots z(1)]}{p[z(k)/z(k-1), \ldots z(1)]}$$

As pointed out, the form of the left hand side is known, and is:

$$p[x(k)/z(k), z(k-1), \ldots z(1)]$$
$$= C \exp -\tfrac{1}{2}[(x(k) - x(k/k))^T P(k/k)^{-1} (x(k) - x(k/k))]$$

On the right hand side,

$$p[z(k)/x(k), z(k-1), \ldots z(1)] = p[z(k)/x(k)]$$
$$= C \exp -\tfrac{1}{2}[(z(k) - Hx(k))^T R^{-1} (z(k) - Hx(k))]$$
$$p[x(k)/z(k-1), \ldots z(1)]$$
$$= C \exp -\tfrac{1}{2}[(x(k) - \phi x(k-1/k-1))^T P(k/k-1)^{-1}$$
$$(x(k) - \phi x(k-1/k-1))]$$

$$p[z(k)/z(k-1), \ldots z(1)]$$
$$= C \exp -\tfrac{1}{2}[(z(k) - H \phi x(k-1/k-1))^T \quad (HP(k/k-1)H^T + R)^{-1}$$
$$(z(k) - H \phi x(k-1/k-1))]$$

Defining parameters of the left hand side distribution may be obtained from the right hand side by matching terms [26]. The Kalman filter equations are obtained, which define the left hand side distribution parameters recursively, and one has

$$x(k)/z(k), z(k-1), \ldots . z(1) \sim N[x(k/k), P(k/k)]$$

in terms of known quantities.

In Anderson [1], it is proved that

$$x \sim N[\mu, \Sigma] \quad \Rightarrow \quad s = Dx \sim N[D\mu, D\Sigma D^T]$$

The theorem includes the cases where x may have either a non-singular or singular distribution and D may be nonsingular or of rank less than the dimension of s.

Invoking the theorem, the optimal estimate of s is seen to be $Dx(k/k)$. Hence the optimal estimate of a linear combination of the states is the same linear combination of the optimal estimate of the states.

APPENDIX II

THE KALMAN FILTER AND ESTIMATION OF A GAUSSIAN MEAN

The following discussion outlines an interpretation of the Kalman filter (discrete case) in which it is considered to be a generalized estimation of a Gaussian population mean. The term "generalized" is used to mean that, contrary to usual assumptions in estimation of the mean, the population to be estimated here will be permitted to move between sampling instants. Movement will be assumed to consist of both a prescribed component and a random component.

The analogy to be described permits achieving an intuitive feel for rates of convergence, effects of uncertainty in initial conditions, and effects of random signal process excitation. Such an intuitive feel for filter behavior is frequently difficult to obtain from conventional formulation of the estimation problem.

The univariate case will be considered first, for simplicity. The same analogy, however, carries over to the multivariate case, and will be considered in brief detail later.

"No Dynamics"Situation

To begin with, the classical mean estimation procedure will be written in recursive form. Consider a scalar random variable Z where

$$Z \sim N(x, r)$$

x = unknown population mean

r = known population variance

If $\bar{Z}$ is the sample mean of a sample of size n from the population, then

$$\bar{Z} \sim N(x, p) \qquad \text{where } p = \frac{r}{n}$$

Since the sample mean is the best (minimum variance) estimate of the mean of a Gaussian population [17], $\hat{x} = \bar{Z}$.

Let $z(1), z(2), \ldots . z(k)$ denote sample values of the random variable Z.

Then

$$\hat{x}(k) = \frac{1}{K}\sum_{i=1}^{K} z(i) = \hat{x}(k-1) + \frac{1}{K}\,[z(k) - \hat{x}(k-1)]$$

$$p(k) = \frac{p(1)}{k} = (1 - \frac{1}{k})p(k-1) \qquad k = 2, 3, \ldots\ldots$$

If $b(k) \triangleq \frac{1}{k}$, then

$$\hat{x}(k) = \hat{x}(k-1) + b(k)\,[z(k) - \hat{x}(k-1)]$$

$$p(k) = [1 - b(k)]\,p(k-1) \qquad k = 2, 3, \ldots\ldots$$

It will be useful to note that as each new sample value is considered, the weightings given it and the previous best estimate are in inverse proportion to their respective variances. For example, at the k th stage above,

$$\hat{x}(k) = \frac{k-1}{k}\hat{x}(k-1) + \frac{1}{k}\,z(k)$$

$$\text{Var}\ \hat{x}(k-1) = \frac{r}{k-1}$$

$$\text{Var}\ z(k) = r$$

$$\text{Var}\ \hat{x}(k) = \frac{r}{k} = \frac{\frac{r}{k-1} \cdot r}{r + \frac{r}{k-1}}$$

$$\text{Weighting given to sample value} = \frac{\frac{r}{k}}{r} = \frac{1}{k}$$

$$\text{Weighting given to previous best estimate} = \frac{\frac{r}{k}}{\frac{r}{k-1}} = \frac{k-1}{k}$$

An alternate view is to note that weightings given each sample value and the previous best estimate are proportional to their relative information content, the reciprocal of their relative uncertainties. From this point of view, the information content of a current best estimate (the reciprocal of its variance) is the sum of the information contents of the sample and previous estimate.

Next, the behavior of a Kalman filter of the following description will be compared with that of the sequential estimation procedure above.

The restrictions that the transition matrix (a scalar here) be unity and that there be no signal process excitation will be relaxed later.

$$x = \text{scalar} \qquad H = h = 1$$
$$\Phi = \phi = 1 \qquad R = r$$
$$Q = q = 0 \qquad P = p$$

The four equations describing filter behavior are repeated below for reference as desired.

$$P(k/k-1) = \Phi P(k-1/k-1)\,\phi^T + Q$$
$$B(k) = P(k/k-1)H^T[HP(k/k-1)H^T + R]^{-1}$$
$$x(k/k) = \Phi x(k-1/k-1) + B(k)[z(k) - \phi x(k-1/k-1)]$$
$$P(k/k) = [I - B(k)H]P(k/k-1)$$

Three cases will be discussed, outlining filter behavior under three different levels of uncertainty in initial estimates (a priori input to the filter). Single subscripts will be used in the present section since there is no ambiguity under the conditions assumed, i.e. $p(k/k) = p(k/k-1)$.

Case 1. Great uncertainty in initial estimate of x. Let $p(0) = ar$ where a = large number

$$b(1) = ph(hph^T + r)^{-1} = \frac{ar}{(a+1)r} = \frac{a}{a+1} \approx 1$$
$$p(1) = (1 - b(1)h)p(0) = (1 - \frac{a}{a+1})ar = \frac{a}{a+1} \approx r$$
$$b(2) \approx \frac{r}{2r} = \tfrac{1}{2} \qquad \Longrightarrow \qquad b(k) = \frac{1}{k}$$
$$p(2) \approx (1 - \tfrac{1}{2})r = \frac{r}{2} \qquad p(k) = \frac{r}{k}$$

Case 2. Uncertainty in initial estimate of x approximately equal to uncertainty in the observation process, i.e. $p(0) = r$.

$$b(1) = \frac{r}{2r} = \tfrac{1}{2} \qquad \Longrightarrow \qquad b(k) = \frac{1}{k+1}$$
$$p(1) = [1 - b(1)]p(0) = \frac{r}{2} \qquad p(k) = \frac{r}{k+1}$$

Case 3. Uncertainty in initial estimate of x less than uncertainty in observation process.

Let $p(0) = \frac{r}{\ell}$ where ℓ is some positive integer.

$$b(1) = \frac{r}{\ell}\left(\frac{r}{\ell} + r\right)^{-1} = \frac{r}{\ell} \quad \frac{\ell}{r(1+\ell)} = \frac{1}{1+\ell}$$

$$p(1) = (1 - b(1))p(0) = \left(1 - \frac{1}{1+\ell}\right)\frac{r}{\ell} = \frac{\ell}{1+\ell}\frac{r}{\ell} = \frac{r}{1+\ell}$$

$$b(2) = \frac{r}{1+\ell}\left(\frac{r}{1+\ell} + r\right)^{-1} = \frac{1}{1+\ell} \quad \frac{1+\ell}{2+\ell} = \frac{1}{2+\ell}$$

$$p(2) = (1 - b(2))p(1) = \left(1 - \frac{1}{2+\ell}\right)\frac{r}{1+\ell} = \frac{r}{2+\ell}$$

$$b(k) = \frac{1}{k+\ell}$$

$$p(k) = \frac{r}{k+\ell}$$

In the Kalman filter for the particular process just described, effects of varying degrees of uncertainty in the initial estimate are isolated and clearly observed. It can be seen that for great uncertainty in the initial estimate, the filter behaves asymptotically as the sequential estimation procedure outlined, i.e. $p(k)$ and $b(k)$ $(k = 2, 3, \ldots n)$ decrease from r and 1 respectively as $\frac{1}{k}$. For uncertainty in the initial estimate equal to or less than uncertainty imposed by the observation process such that $p(0) = \frac{1}{\ell} r$ (ℓ = positive integer), then the filter data weighting and convergence rate are the same as the data weighting and convergence rate of the sampling estimation scheme from its $(\ell+1)$th sample. In other words, a priori information may be regarded as providing equivalent additional observations of the process to be estimated.

It is thus clearly shown that under the conditions stated, p and b decrease asymptotically as $\frac{1}{k}$. Interpolating between the assumed rational values of the ratio $\frac{r}{p(0)}$ yields a gain schedule $b(k) = \frac{1}{k+a}$ $\left(a = \frac{r}{p(0)}\right)$ for the filter in the simple situation being considered.

Unexcited Dynamic Situation

Next, a more general univariate situation will be considered. In the Kalman filter representation, the restriction above that $\phi = 1$ will be relaxed. The corresponding situation in the sequential estimation procedure will be that during the sampling process, the population mean moves about in a prescribed fashion, i.e. $x(k+1) = \phi x(k)$. As pointed out above, the sequential estimation procedure forms a weighted sum of the previous best estimate and the new data (sample value), and specifies a new estimate variance equal to the "parallel combination" of the sample and previous optimal estimate variances.

The essential difference between the present case with relaxed restrictions on ϕ and the case previously discussed is simply that just as the population mean is moved between samples, the previous best estimate must be adjusted by ϕ prior to adding in effects of the new sample value.

A double-subscript notation will be helpful here. Let $x(k/k-1) \triangleq$ The optimal estimate of x at sample time k given samples up to sample time $k-1$.

Since $x(k) = \phi x(k-1)$, $\hat{x}(k/k-1) = \phi \hat{x}(k-1/k-1)$

and since $\hat{x}(k-1/k-1) \sim N(x(k-1), p(k-1/k-1))$,

$\hat{x}(k/k-1) \sim N(x(k), \phi^2 p(k-1/k-1))$.

Hence,

$$p(k/k) = \frac{r \phi^2 p(k-1/k-1)}{r + \phi^2 p(k-1/k-1)} = \frac{rp(k/k-1)}{r + p(k/k-1)}$$

$$= b(k)r$$

The resulting gains with which $\hat{x}(k/k-1)$ and $z(k)$ are added are

$$\frac{p(k/k)}{p(k/k-1)} \quad \text{and} \quad \frac{p(k/k)}{r} .$$

No additional discussion of the Kalman filter is required for this situation, where ϕ is not constrained to be unity. If the filter a priori variance $p(0)$ equals $p(k/k-1)$ of the sequential estimation procedure for any k, the filter will behave from the beginning just

as the estimation scheme does from the k th stage on.

Randomly Excited Dynamics

The situation may be made still more general by relaxing the restriction in A that $q = 0$. In the sequential estimation procedure, the interpretation is that not only will the population mean move in a prescribed fashion between sampling instants, but will have an additional random motion imparted to it by a gaussian excitation independent from sample to sample.

Again, no additional discussion of the Kalman filter is required. Further, treatment of the present situation in the sequential estimation procedure is a direct extension from B.

The first few steps are

$$p(1) = r$$

$$b(1) = 1$$

$$p(2/1) = \phi^2 r + q$$

$$p(2/2) = \frac{(\phi^2 r + q)r}{(\phi^2 + 1)r + q}$$

$$b(2) = \frac{p(2/2)}{r} = \frac{\phi^2 r + q}{(\phi^2 + 1)r + q}$$

etc.

The steps in general are

$$p(k/k-1) = \phi^2 p(k-1/k-1) + q$$

$$p(k/k) = \frac{p(k/k-1)r}{p(k/k-1) + r}$$

$$b(k) = \frac{p(k/k-1)}{p(k/k-1) + r}$$

The general updating steps agree with those of the Kalman filter. Again, if the filter a priori variance $p(0)$ equals $p(k/k-1)$ of the sequential estimation procedure for some k, the same remark as in B is valid. For the present situation, p does not continue indefinitely to decrease, but reaches a non-zero steady state value. This steady state value of p and the corresponding steady state gain may be computed by setting $p(k+1/k+1) = p(k/k)$ and solving.

The Multivariate Case

Discussion of the multivariate case will be limited to situations where $H = I$. Though this excludes cases of major interest where the Kalman filter has been employed, the additional detail required would obscure the comparison sought here.

Under the frequent assumptions of diagonal $P(0)$ and R matrices, the case where $\Phi = I$ and $Q = 0$ (no dynamics) really amounts to n uncoupled problems of the type covered above, and is of little additional interest. Ho has shown [15] that P decreases asymptotically as $\frac{1}{k}$ for this case without the assumption of a diagonal $P(0)$.

The randomly excited dynamic situation is of more general interest and is a direct extension of the approach in the univariate case. As before, if the filter a priori covariance matrix $P(0) = P(k/k-1)$ of the sequential estimation procedure for any k, its effect may be regarded as supplying k equivalent additional observations to the filter. Adjustment of the optimal estimate covariance between samples is:

$$P(k/k-1) = \Phi P(k-1/k-1)\, \Phi^T + Q.$$

Updating of the covariance matrix and computation of the new gain is also a direct extension:

Univariate case:	Multivariate case:
$p(k/k) = \frac{p(k/k-1)\, r}{p(k/k-1) + r}$	$P(k/k) = P(k/k-1)(P(k/k-1) + R)^{-1}R$
$b(k) = \frac{p(k/k)}{r}$	$B(k) = P(k/k)R^{-1}$

A check with the filter equations above will disclose that this is exactly the same updating sequence as in the Kalman filter.

APPENDIX III

COMPUTER PROGRAM FOR ESTIMATING SINS LATITUDE ERROR

(PROGRAM WRITTEN IN FORTRAN 60 FOR THE CDC 1604)

```
      PROGRAM NAVERR
      DIMENSION PHI(6,6),D(6),H(6),XST(6),X10(6),
     1P(6,6),P10(6,6),G(6),TT(6,6),TV(6),TQ(6,6) ,X(6)
     2,XP(500),YP(500),ITITLE(12),YSTP(500),ZK(30),XK(30)
C    THIS PROGRAM TESTS THE APPLICATION OF TIME-DOMAIN ESTIMATION
C    TECHNIQUES TO THE PROBLEM OF ERROR DETERMINATION IN INERTIAL
C    NAVIGATION SYSTEMS.  MORE SPECIFICALLY, IT WILL PROVIDE A RUNNING
C    LEAST-SQUARES-ERROR ESTIMATE OF SYSTEM LATITUDE ERROR AND WILL
C    SUPPLY THE CORRESPONDING VARIANCE OF ITS ESTIMATE.  IN THE PRESENT
C    PROGRAM, IT IS ASSUMED THAT AN UPDATED ESTIMATE OF LATITUDE ERROR
C    IS DESIRED HOURLY, AND THAT NOISY OBSERVATIONS OF LATITUDE ERROR
C    ARE AVAILABLE EVERY NR HOURS, NR TO BE SPECIFIED.  QUALITY OF THE
C    OBSERVATIONS OF LATITUDE ERROR IS CONSIDERED IN OBTAINING RELATIVE
C    WEIGHTING OF EACH OBSERVATION AND EXTRAPOLATED PRIOR KNOWLEDGE OF
C    THE SYSTEM ERROR.  FOR SIMPLICITY, THIS PROGRAM ASSUMES EACH OBSER-
C    VATION USED TO BE OF THE SAME QUALITY, BUT SUCH AN ASSUMPTION IS
C    NOT REQUIRED BY THE METHOD.
     51 FORMAT (3I10)
     52 FORMAT (2F10.4)
     53 FORMAT(8(F10.4,5X))
     54 FORMAT(I10,5X,3(1PE10.3,5X))
     55 FORMAT(F10.4)
     56 FORMAT (6A8)
     57 FORMAT (5X,27H STATES AND STATE ESTIMATES  )
     58 FORMAT (5X,21H  COVARIANCE MATRIX P   )
     59 FORMAT(1H0,I3,35H PIECES OF DATA HAVE BEEN PROCESSED )
     60 FORMAT(10H0   HOURS ,5X,10H LAT ERROR,4X,12H EST LAT ERR,
       14X,12H EST ERR VAR)
     61 FORMAT(1H1)
        NU=1220903125
        READ 51,N,NR,NT
```

```
C  N IS ORDER OF ERROR GENERATING MODEL
C  NR IS NUMBER OF HOURS BETWEEN RECEIPT OF NRW DATA
C  NT IS NUMBER OF MEASUREMENTS PROCESSED IN PROGRAM SIMULATION
C  THUS NR*NT IS TOTAL NUMBER OF HOURS COVERED BY SIMULATION
      READ 52,Q,R
      DO 1 I=1,N
      X10(I)=0.
    1 READ 53,(PHI(I,J),J=1,N)
      READ 53,(D(I),I=1,N)
      READ 53,(H(I),I=1,N)
      CALL STEADYA(PHI,Q,P10,D,N)
      X(1) =2.
      X(2)=8.
      X(3)=0.
      Y=0.
      DO 111 I=1,N
  111 Y=Y+H(I)*X(I)
      CALL RNDEV(NU,V)
      V=V*SQRTF(R)
      Z=Y+V
      KP = 1
      PRINT 61
      DO 2 K=1,NT
      ZK(K)=Z
      XK(K)=KP
C  WEIGHTING COMPUTATION.  THESE COMPUTATIONS DO NOT DEPEND ON OBSER-
C  VATIONS EXCEPT FOR THEIR QUALITY.  HENCE, IF ALL OBSERVATIONS ARE
C  ASSUMED TO BE OF THE SAME QUALITY, COMPUTATIONS HERE MAY BE DONE
C  OFF LINE BEFOREHAND AND THE RESULTING WEIGHTING STORED IN A LIST.
      T=R
      DO 3 I=1,N
      G(I)=0.
      DO 3 J=1,N
      T=T+ H(I)*P10(I,J)*H(J)
    3 G(I)=G(I)+P10(I,J)*H(J)
      Z10=0.
```

```
      DO 4 I=1,N
      G(I)=G(I)/T
    4 Z10=Z10+H(I)*X10(I)
C   DETERMINATION OF OPTIMAL (LEAST SQUARES) ESTIMATE
      ZTIL = Z-Z10
      DO 5 I=1,N
    5 XST(I)=X10(I)+G(I)*ZTIL
C   UPDATING ESTIMATE VARIANCE AFTER RECEIPT OF NEW OBSER-
C   VATION.  P=P10-G*H*P10
      DO 6 I=1,N
      TV(I)=0.
      DO 6 J=1,N
    6 TV(I)=TV(I)+H(J)*P10(J,I)
      DO 7 I=1,N
      DO 7 J=1,N
    7 P(I,J)=P10(I,J)-G(I)*TV(J)
      PRINT 59,K
      PRINT 57
      PRINT 53 (X(I),I=1,N)
      PRINT 53 (XST(I),I=1,N)
      PRINT 58
      DO 14 I=1,N
   14 PRINT 53 (P(I,J),J=1,N)
      DO 2 M=1,NR
      PY=0.
      DO 8 I=1,N
      DO 8 J=1,N
    8 PY=PY+H(I)*P(I,J)*H(J)
      YST=0.
      DO 9 I=1,N
    9 YST=YST+H(I)*XST(I)
C   OUTPUT OF ESTIMATE AND VARIANCE
      PRINT 60
      PRINT 54,KP,Y,YST,PY
      YP(KP) = Y
      YSTP(KP) = YST
      XP(KP) = KP
```

```
C  THE PRESENT LOOP(M) UPDATES ESTIMATES AND VARIANCE DURING PERIODS
C  BETWEEN RECEIPT OF DATA.  AS PROGRAMMED, HOURLY UPDATING OF
C  ESTIMATES AND VARIANCE IS PERFORMED.  NEW DATA IS ASSUMED AVAILABLE
C  EVERY NR HOURS, AND IS INCORPORATED INTO ESTIMATES AS AVAILABLE.
      DO 11 I=1,N
      X10(I)=0.
      DO 11 J=1,N
      TT(I,J)=0.
      X10(I)=X10(I)+PHI(I,J)*XST(J)
      DO 11 L=1,N
   11 TT(I,J)=TT(I,J)+P(I,L)*PHI(J,L)
      DO 12 I=1,N
      XST(I)=X10(I)
      DO 12 J=1,N
      P10(I,J)=0.
      TQ(I,J)=Q*D(I)*D(J)
      DO 12 L=1,N
   12 P10(I,J)=P10(I,J)+PHI(I,L)*TT(L,J)
      DO 13 I=1,N
      DO 13 J=1,N
      P10(I,J)=P10(I,J)+TQ(I,J)
   13 P(I,J)=P10(I,J)
      CALL RNDEV(NU,W)
      W=W*SQRTF(Q)
      DO 113 I=1,N
      TV(I)=0.
      DO 113 J=1,N
  113 TV(I)=TV(I)+PHI(I,J)*X(J)+D(I)*W
      DO 112 I=1,N
      X(I)=TV(I)
  112 CONTINUE
      Y=0.
      DO 15 I=1,N
   15 Y=Y+H(I)*X(I)
      CALL RNDEV (NU,V)
```

```
      V=V*SQRTF(R)
      Z=Y+V
      KP = KP + 1
    2 CONTINUE
      NP = NR*NT
      MOD = 1
      LAB = 4H Y
      READ 56, (ITITLE(I), I=1,6)
      READ 56, (ITITLE(I), I=7,12    )
      CALL DRAW (NP,XP,YP  ,MOD,0,LAB,ITITLE,0,5.,2,0,2,0,6,4,0,LAST)
      MOD = 3
      LAB = 4H YST
      CALL DRAW (NP,XP,YSTP,MOD,0,LAB,ITITLE,0,5.,2,0,2,0,6,4,0,LAST)
      MOD=1
      LAB=4H YST
      READ 56, (ITITLE(I), I=1,6)
      READ 56, (ITITLE(I), I=7,12    )
      CALL DRAW (NP,XP,YSTP,MOD,0,LAB,ITITLE,0,5.,2,0,2,0,6,4,0,LAST)
      MOD=3
      LAB=8H
      CALL DRAW (NT,XK,ZK  ,MOD,2,LAB,ITITLE,0,5.,2,0,2,0,6,4,0,LAST)
      END

      SUBROUTINE STEADYA (PHI,Q,P10,D,N)
      DIMENSION PHI(6,6),D(6),P10(6,6),TQ(6,6),TP(6,6),TT(6,6)
C  THIS SUBROUTINE CALCULATES THE STEADY STATE COVARIANCE MATRIX OF
C  THE STATES OF A LINEAR DYNAMIC SYSTEM EXCITED BY W-ITE NOISE OF
C  SPECIFIED VARIANCE.  THE DYNAMIC SYSTEM IS ASSUMED TO BE  ABSO-
C  LUTELY STABLE  AND TO HAVE ONLY A SINGLE INPUT.  STEADY STATE IS
C  APPROXIMATED BY AN ALGORITHM WHICH IS EQUIVALENT TO STEPPING THE
C  DIFFERENCE EQUATION P(K+1)=PHI*P(K)*PHI+D*Q*D    1024 TIMES.
   50 FORMAT(1X,8E10.2)
   51 FORMAT(20X,4H PHI )
   52 FORMAT(10X,31H STEADY-STATE COV MATRIX OF SGP )
      QQ=50.*Q
      DO 1 I=1,N
```

```
      DO 1 J=1,N
    1 TP(I,J)=PHI(I,J)
      DO 2 I=1,N
      DO 2 J=1,N
    2 TQ(I,J)=QQ*D(I)*D(J)
      PRINT 51
      DO 3 I=1,N
    3 PRINT 50,(TP(I,J),J=1,N)
      DO 10 KK=1,10
      DO 4 I=1,N
      DO 4 J=1,N
      TT(I,J)=0.
      DO 4 K=1,N
    4 TT(I,J)=TT(I,J)+TQ(I,K)*TP(J,K)
      DO 5 I=1,N
      DO 5 J=1,N
      DO 5 K=1,N
    5 TQ(I,J)=TQ(I,J)+TP(I,K)*TT(K,J)
      DO 6 I=1,N
      DO 6 J=1,N
      TT(I,J)=0.
      DO 6 K=1,N
    6 TT(I,J)=TT(I,J)+TP(I,K)*TP(K,J)
      DO 10 I=1,N
      DO 10 J=1,N
   10 TP(I,J)=TT(I,J)
      DO 7 I=1,N
      DO 7 J=1,N
    7 P10(I,J)=TQ(I,J)
      PRINT 52
      DO 8 I=1,N
    8 PRINT 50,(P10(I,J),J=1,N)
      END
      END
```

APPENDIX IV

(PROGRAM WRITTEN IN FORTRAN 63 FOR CDC 1604)

```
-COOP,,POPE J W BOX P, I/1/O/49/S/1S/2S/E/45=54,10,9000,4.
-FTN,L,E,P,A.
      PROGRAM MODEL                                                    0003
C  THIS PROGRAM IS DESIGNED TO OBTAIN THE BEST AUTOREGRESSIVE
C  MODEL OF A SPECIFIED ORDER FOR A RANDOM TIME SERIES READ
C  IN AS DATA.  SAMPLES OF THE RANDOM TIME SERIES INPUT AS DATA
C  ARE ASSUMED TO BE IN BLOCKS OF 4000, EACH BLOCK IDENTIFIED
C  WITH A UNIQUE 16 DIGIT OCTAL IDENTIFIER.  ORDER OF THE AUTO-
C  REGRESSIVE MODEL IS N.  EACH KDEL TH SAMPLE IN A TOTAL RUN
C  OF NK SAMPLES OF A BLOCK IS USED IN OBTAINING THE MODEL.
C  COEFFICIENTS OF THE MODEL OBTAINED ARE PRINTED EACH 50
C  SAMPLES DURING PROCESSING.  AT COMPLETION, ADDITIONAL
C  GRAPHICAL OUTPUT  IS PROVIDED TO FURTHER DESCRIBE MODEL
C  OBTAINED.  GRAPHICAL OUTPUT CONSISTS OF A PLOT OF Z-PLANE
C  POLES OF THE MODEL, COMPARATIVE ACF PLOTS GENERATED FROM
C  MODEL AND DATA, AND COMPARATIVE PSD PLOTS GENERATED FROM
C  MODEL AND DATA.
C  PROVISION IS INCLUDED FOR SPECIFYING ORDER OF MODEL NUMER-
C  ATOR M.  FOR AUTOGRESSIVE MODEL, SET M=1.  FOR M GREATER
C  THAN ONE, PROGRAM OBTAINS DENOMINATOR COEFFICIENTS OF A
C  DISCRETE LINEAR FILTER MODEL WITH M TH ORDER NUMERATOR.
                                                                       0006
      DIMENSION IBLOCK(1001),KDATA(4000,2),ZD(4000),P(12,12),          0007
     1P10(12,12),Z(12),G(12),PHEE(12),TT(12)                           0008
     2,XR(13),RR(12),RI(12),CONV(12)                                   0009
     3,ITITLE(12),LAS(12,10),LABS(20)                                  000
      COMMON KDATA,IBLOCK                                              0010
   50 FORMAT(6I10,E10.3)
   51 FORMAT(38H1DIMENSION OF SIGNAL PROCESS MODEL IS ,I5)              0012
   52 FORMAT (15H ERROR IN DATA )                                       0013
   53 FORMAT(26H SAMPLE AVERAGE REMOVED = ,   E10.3)                    0014
   54 FORMAT (   40X, 6H PHEE )                                         0015
```

```
   55 FORMAT(12E10.2)                                                    0016
   56 FORMAT(30X,26H PSEUDOCOVARIANCE OF PHEE    )                       0017
   57 FORMAT (27H NR OF SAMPLES PROCESSED = ,I5,2H / ,I2)                0018
   58 FORMAT(36H FOL ARE ROOT REAL PARTS, IMAG PARTS )                   0019
   59 FORMAT(12E10.2)                                                    0020
   60 FORMAT(O16)                                                        0021
   61 FORMAT(27H NUMBER BLOCKS PROCESSED = ,I2)                          0022
   62 FORMAT(1H1)                                                        0023
   63 FORMAT(6A8)                                                        0024
   64 FORMAT(22HORDER OF NUMERATOR IS ,I3)

      READ 50,M,N,NK,KDEL,NB,NTAU,DEL
C  KDEL MUST NOT BE ZERO                                                0026
C  N IS ORDER OF MODEL, NK/KDEL IS NR OF SAMPLES PROCESSED              0027
C  NB IS NUMBER OF BLOCKS TO BE PROCESSED                               0028
C  NTAU IS NUMBER OF ACF LAGS TO BE COMPUTED.
C  NTAU COMPUTED FROM PSD WINDO WIDTH DESIRED
C  FROM RELATION  NTAU=1/2*DEL*FCOMB.
C  DEL IS SAMPLING INTERVAL IN SECONDS.
      PRINT 51, N                                                       0029
      PRINT 57, NK,KDEL                                                 0030
      PRINT 61,NB                                                       0031
      MAX = 4000                                                        0032
      KLIST = 1                                                         0033
      LAS(1,3)=4H 0    $  LAS(2,3)=4H 139    $  LAS(3,3)=4H 278         0036
      LAS(4,3)=4H 417 $  LAS(5,3)=4H 556    $  LAS(6,3)=4H 695         0037
      LAS(7,3)=4H 833 $  LAS(8,3)=4H        $  LAS(9,3)=4H             0038
      LAS(10,3)=4H             $  LAS(11,3)=4H          $  LAS(12,3)=4H 0039
      LAS(13,3)=4H             $  LAS(14,3)=4H          $  LAS(15,3)=4H 0040
      LAS(16,3)=4H             $  LAS(17,3)=4H          $  LAS(18,3)=4H 0041
      LAS(19,3)=4H             $  LAS(20,3)=4H                         0042
      LABS(1)=8H 1    1   $  LABS(2)=8H 2    2  $ LABS(3)=8H 3    3   0043
      LABS(4)=8H 4    4   $  LABS(5)=8H 5    5  $ LABS(6)=8H 6    6   0044
      LABS(7)=8H 7    7   $  LABS(8)=8H 8    8  $ LABS(9)=8H 9    9   0045
      LABS(10)=8H 10  10                                                0046
      DO 200 KSIG=1,NB                                                  0047
```

```
      READ 60, IDENT                                                    0048
      READ 63,(ITITLE(I),I=1,6)                                         0034
      READ 63,(ITITLE(I),I=7,12)                                        0035
   18 CALL DATA (IDENT,MAX,KLIST,KFLAG)                                 0049
      IF(KFLAG) 1,2,1                                                   0050
    1 PRINT 52                                                          0051
      GO TO 100                                                         0052
    2 CONTINUE                                                          0053
      ZAV=0.                                                            0054
      DO 3 K=1,MAX                                                      0055
      ZD(K)=KDATA(K,1)                                                  0056
    3 ZAV=ZAV+ZD(K)                                                     0057
      XMAX=MAX                                                          0058
      ZAV=ZAV/XMAX                                                      0059
      PRINT 53,ZAV                                                      0060
      DO 4 K=1,MAX                                                      0061
    4 ZD(K)=ZD(K)-ZAV                                                   0062
C   INITIAL ESTIMATE AND VARIANCE OF PHEE                               0063
      DO 15 I=1,N                                                       0064
      PHEE (I) =0.                                                      0065
      DO 15 J=1,N                                                       0066
   15 P10(I,J)=0.                                                       0067
      PHEE(N)=1.
      DO 5 I=1,N                                                        0069
    5 P10(I,I)=100.                                                     0070
C   NOW LOOPS TO ESTIMATE PHEE AND UPDATE P.                            0071
C   PHEE ESTIMATES ARE PRINTED EACH 50 ITERATIONS                       0072
C   CONSTRUCTION OF INITIAL Z VECTOR.  NOTE THAT Z(N)                   0073
C   IS MOST RECENT DATA.                                                0074
      NS=KDEL*N                                                         0075
      DO 6 K=1,NS,KDEL                                                  0076
    6 Z(K)=ZD(K)                                                        0077
      PRINT 64,M
      K=NS+KDEL
      PRINT 54                                                          0080
    7 CONTINUE                                                          0081
```

```
      PRINT 55, (PHEE(I),I=1,N)                                    0082
    8 DO 9 JJ=1,50                                                 0083
C   GAIN COMPUTATION                                               0088
C   EQUATION IS G=P10Z(ZP10Z+RO)(INV)                              0089
      RO=1.
      A=RO                                                         0090
      DO 10 I=1,N                                                  0091
      G(I)=0.                                                      0092
      DO 10 J=1,N                                                  0093
      A=A+Z(I)*P10(I,J)*Z(J)                                       0094
   10 G(I)=G(I)+P10(I,J)*Z(J)                                      0095
      Z10=0.                                                       0096
      DO 11 I=1,N                                                  0097
      G(I)=G(I)/A                                                  0098
   11 Z10=Z10+PHEE(I)*Z(I)                                         0099
      ZTIL=ZD(K)-Z10                                               0100
C   UPDATING PHEE ESTIMATE                                         0101
      DO 12 I=1,N                                                  0102
   12 PHEE(I)=PHEE(I)+G(I)*ZTIL                                    0103
C   UPDATING P10                                                   0104
C   EQUATION IS  P10=PHI(P10-GZP10)PHI                             0105
C    PHI IS IDENTITY MATRIX                                        0106
      DO 13 I=1,N                                                  0107
      TT(I)=0.                                                     0108
      DO 13 J=1,N                                                  0109
   13 TT(I)=TT(I)+Z(J)*P10(J,I)                                    0110
      DO 14 I=1,N                                                  0111
      DO 14 J=1,N                                                  0112
   14 P10(I,J)=P10(I,J)-G(I)*TT(J)                                 0113
C   SHIFT OF Z VECTOR                                              0114
      DO 19 JN=1,M
      DO 17 I=2,N                                                  0115
      L=I-1                                                        0116
   17 Z(L)=Z(I)                                                    0117
      Z(N)=ZD(K)                                                   0118
      K=K+KDEL                                                     0084
```

```
      IF(K.GT.NK) 100,19                                                0085
   19 CONTINUE                                                          0086
    9 CONTINUE                                                          0119
      GO TO 7                                                           0120
  100 CONTINUE                                                          0121
      PRINT 55, (PHEE(I),I=1,N)                                         0122
      NN=N+1                                                            0123
      XR(1)=-1.                                                         0124
      DO 20 I=2,NN                                                      0125
      J=NN-I+1                                                          0126
   20 XR(I)=PHEE(J)                                                     0127
      CALL RTPLSUB(N,XR,RR,RI,CONV)
      PRINT 58                                                          0129
      PRINT 59,(RR(J), J=1,N)                                           0130
      PRINT 59, (RI(J), J=1,N)                                          0131
      PRINT 56                                                          0132
      DO 16 I=1,N                                                       0133
   16 PRINT 55,(P10(I,J),J=1,N)                                         0134
      PRINT 62                                                          0135
      IPASS=4
      CALL ZPLOT(RR,RI,N, 3  ,KSIG,IPASS,LAS,LABS,ITITLE)               0147
      CALL RTAU(PHEE,ZD,N,NK,KDEL,NTAU,DEL)
  200 CONTINUE                                                          0149
         END

      SUBROUTINE DATA(IDENT,MAX,KLIST,KFLAG)
      DIMENSION IBLOCK(1001),KDATA(4000,2)
      COMMON KDATA, IBLOCK
C     CALLING INSTRUCTIONS FOR SUBROUTINE DATA
C          ARG 1.  IDENT IS A 16 OCTAL DIGIT IDENTIFYING NUMBER
C               SUPPLIED BY THE 160 PROGRAM WHEN TAPE WAS MADE.
C               THE CALLING PROGRAM MUST SUPPLY THIS NUMBER IN ORDER FOR
C               THE CORRECT BLOCK OF DATA TO BE RECALLED.
C          ARG 2.  MAX IS THE NUMBER OF SAMPLES PER BLOCK AND MUST BE LESS
C                    THAN 4000 DEC.
C          ARG 3.  KLIST IS EITHER 1 OR 2 AND DECIDES WHICH SIDE OF
```

```
C              KDATA YOU WANT THE DATA TO BE UNPACKED INTO
C          ARG 4.  KFLAG IS AN ERROR FLAG RAISED BY DATA. IF =0,NO ERROR
C          LOGICAL UNIT 1 IS USED BY DATA TO FIND BLOCK.
C          EACH TIME DATA IS CALLED IT FINDS THE DESIRED BLOCK ON THE TAPE,
C             AND UNPACKS IT INTO EITHER KDATA(M,1) OR KDATA(M,2)
C          IN THE CALLING PROGRAM DIMENSION KDATA(4000,2),  AND
C              DECLARE KDATA COMMON.
C     KDATA IS THE OUTPUT LIST AND IS REFERENCED BY KLIST
C     SUBROUTINE DATA CALLS ON SR UNPACK AND SR FINDIT
C     AFTER UNPACK THERE EXISTS ONE 1604 WORD/160 WORD
C     ASSIGNMENT NOS USED IN DATA,822,823,824,825,828,829
C     SPARE ASSIGNMENTS 826,827
  824 FORMAT(48H  ERROR OCCURRED IN FINDIT SUBROUTINE AT HEADER )
  825 FORMAT(50X,O16)
  828 FORMAT(33H  I HAVE UNPACKED DATA HEADED BY )
      CALL FINDIT(IDENT,MAX,IFLAG)
      IF(IFLAG) 822,823,822
  822 PRINT 824 $ PRINT 825,IDENT $KFLAG=1$ GO TO 829
  823 JMAX=MAX/4+1
      CALL UNPACK (IBLOCK,JMAX,KLIST,KDATA(1,KLIST))
      KFLAG=0$ PRINT 828 $ PRINT 825,IDENT
      PRINT 825,IDENT
  829 CONTINUE
      END
      SUBROUTINE FINDIT(IDENT,MAX,IFLAG)
      DIMENSION KDATA(4000,2),IBLOCK(1001)
      COMMON KDATA,IBLOCK
  814 FORMAT(53H  A PARITY ERROR WAS DETECTED BUT RUN WAS NOT STOPPED)
  815 FORMAT(45H  PARITY ERROR OCCURRED AT HEADER AS FOLLOWS )
  816 FORMAT(33X,O16)
  818 FORMAT (35H I HAVE LOCATED HEADING AS FOLLOWS )
  820 FORMAT (38H  UNABLE TO LOCATE HEADING AS FOLLOWS )
      MAX1=MAX/4 + 1
      ASSIGN 811 TO JUMP
      IPAR=0
  805 BUFFER IN (1,1)(IBLOCK(1),IBLOCK(MAX1))
```

```
  806 IF(UNIT,1) 806,807,808,810
  807 IF(IDENT-IBLOCK(1))805,813,805
  808 GO TO JUMP,(811,812)
  811 REWIND 1$ ASSIGN 812 TO JUMP $ GO TO 805
  810 IPAR=1  $  GO TO 807
  812 REWIND 1$ GO TO 819
  809 PRINT 814$ PRINT 815$ PRINT 816,IDENT$ GO TO 817
  813 IF(IPAR) 809,817,809
  817 IFLAG=0$ PRINT 818$ PRINT 816,IDENT $ GO TO 821
  819 PRINT 820
      PRINT 816,IDENT
      IFLAG=1
C     ASSIGNMENT NOS USED HERE,805,807,808,809,810,811,812,813,814,815
C                              816,817,818,819,820,821
  821 CONTINUE
      END
```



```
         IDENT     UNPACK              ARGS IBLOCK,JMAX,KLIST,KDATA
         ENTRY     UNPACK              SUBR UNPACK CALLED BY SR DATA
UNPACK   SLJ       **                  +
         SIU    1  EXIT
         LIU    1  UNPACK              + JMAX=MAX/4+1, MOST=JMAX-1
         LDA    1  0                   GETS ADDR OF CALLING ARGS
         SAL        U1                  +U1 IS JMAX ADDR
         ARS        24                  IBLOCK PACKED IN DATA 1001 WDS
         INA        -1                  +
U1       SAU        AADRS               ADRS OF IBLOCK IN AADRS
         LDA        **                  +FILLED IN BY 3 INSRUCTIONS BACK
         SAU        BADRS               STORE JMAX IN LOOP COUNT
         LDA    1  1                   +GO GET ADRS OF NEXT ARG
         SAU        J4                  +STORE ADRS OF KDATA(1,KLIST)
         INA        1                   ADVANCE ADRS
         SAL        J3                  +STORE ADRS OF KDATA(2,KLIST)
         INA        1
         SAU        J2                  +DITTO FOR KDATA(3,KLIST)
         INA        1
         SAL        J1                  +DITTO FOR KDATA(4,KLIST)
```

```
         INI    1  2                    SET CORRECT EXIT ADRS
         SIU    1  EXIT+1               +STORE IN EXIT INST
         SIL    2  EXIT                 SAVE INDEX 2
         ENI    1  2                    +FIRST DATA WORD IN IBLOCK
         ENI    2  0                    FIRST WORD IN KDATA IS DATA
AADRS    LDA    1  **                   +IBLOCK(J)
         LRS       12                   SHIFT 4TH WORD INTO A REG
J1       QRS       36                   +RIGHT JUST,SIGN EXTEND
         STQ    2  **                   STORE IN KDATA (I+4,KLIST)
         LRS       12                   +SHIFT 3RD WORD INTO Q
         QRS       36                   RIGHT JUST ,SIGN EXTEND
J2       STQ    2  **                   +STORE IN KDATA(I+3,KLIST)
         LRS       12                   SHIFT 2ND WORD INTO A REG
J3       QRS       36                   + RIGHT JUST, SIGN EXTEND
         STQ    2  **                   STORE IN KDATA(I+2,KLIST)
J4       STA    2  **                   +1ST WORD NOW RT.JUST +SIGN EXT
         INI    2  4                    I=I+4
BADRS    ISK    1  **                   +ISK ON JMAX
         SLJ       AADRS                J=J+1 REPEAT LOOP
EXIT     ENI    1  **                   +RESTORE INDEX 1
         ENI    2  **                   RESTORE INDEX 2
         SLJ       **                   + JUMP OUT
         END                            SUBROUTINE UNPACK
      SUBROUTINE ZPLOT (RR,RI,IPTS,J2,J3,IPASS,LAS,LABS,IT)                0261
      DIMENSION RR(20),IT(12),LAS(12,10),X(60),Y(60),LABS(20),RI(20)       0262
      IF(IPASS-4) 9,10,10                                                  0263
   10 GO TO 7                                                              0264
    9 CONTINUE                                                            0265
      IF(IPASS-1)8,7,8                                                    0266
    7 PI=ATANF(1.)*4. $ LAB=4H     $M=2                                   0267
      DO 1 IA=1,60 $ AI=IA $ B=AI*PI*.034                                 0268
      X(IA)=COSF(B)                                                       0269
    1 Y(IA)=SINF(B)                                                       0270
      CALL DRAW(60,X,Y,1,0,LAB,IT,.3,.3,4,4,2,2,8,8,0,LT)                 0271
      DO 2 I=1,12 $ A=I $ B=PI*(A-1.)/6  .                                0272
      X(2)=COSF(B)$X(1)=.1*X(2)$Y(2)=SINF(B)$ Y(1)=.1*Y(2)                0273
```

```
    2 CALL DRAW(2,X,Y,2,0,LAS(I,J2),IT,.3,.3,4,4,2,2,8,8,0,LT)                0274
    8 DO 3 I=1,IPTS,2 $ X(1)=RR(I)$X(2)=RR(I+1)$Y(1)=RI(I)$Y(2)=RI(I+1)       0275
      IF(I+1-IPTS)3,4,6                                                       0276
    6 X(2)=RR(I-1)$Y(2)=RI(I-1)                                               0277
    4 IF(IPASS-3) 3,5,5                                                       0278
    5 M=3 $ GO TO 3                                                           0279
    3 CALL DRAW(2,X,Y,M,3,LABS(J3),IT,.3,.3,4,4,2,2,8,8,0,LT)                 0280
      END                                                                     0281
                                                                              0282
                                                                              0283
      SUBROUTINE RTAU(PHEE,ZD,M,NK,KDEL,NTAU,DEL)
      DIMENSION PHEE(12),ZD(4000),PHI(12,12),R(100),                          0285
     1TQ(12,12),TP(12,12),                                                    0286
     2T(12),TT(12,12),P(12,12),XK(100),YK(100),ITITLE(12)                     0287
     3,X(100),FREQ(100)
   51 FORMAT (2I10)                                                           0288
   52 FORMAT(F10.4)                                                           0289
   53 FORMAT(8F10.4)                                                          0290
   54 FORMAT (8(5X,1PE10.3))                                                  0291
   55 FORMAT(8E10.2)                                                          0292
   56 FORMAT(6A8)                                                             0293
   57 FORMAT (10X,14H COV MATRIX P  )                                         0294
   58 FORMAT (10X, 4H PHI )                                                   0295
      DO 1 I=1,M                                                              0296
      DO 1 J=1,M                                                              0297
    1 PHI(I,J)=0.                                                             0298
      DO 2J=2,M                                                               0299
       I=J-1                                                                  0300
    2 PHI(I,J)=1.                                                             0301
      DO 6 I=1,M                                                              0302
    6 PHI(M,I)=PHEE(I)                                                        0303
      DO 7 I=1,NTAU                                                           0305
    7 XK(I)=I                                                                 0306
      MOD =1                                                                  0307
      LAB=4H RM                                                               0308
      READ 56, (ITITLE(I),I=1,6)                                              0309
```

```
      READ 56, (ITITLE(I),I=7,12)                                    0310
      Q=1.                                                           0311
C NOW TO OBTAIN STEADY STATE CON MATRIX P OF LDS                     0312
      DO 24 I=1,M                                                    0313
      DO 24 J=1,M                                                    0314
      TQ(I,J)=0.                                                     0315
   24 TP(I,J)=PHI(I,J)                                               0316
      TQ(M,M)=Q                                                      0317
      DO 10 KK=1,10                                                  0318
      DO 25 I=1,M                                                    0319
      DO 25 J=1,M                                                    0320
      TT(I,J)=0.                                                     0321
      DO 25 K=1,M                                                    0322
   25 TT(I,J)=TT(I,J)+TQ(I,K)*TP(J,K)                                0323
      DO 26 I=1,M                                                    0324
      DO 26 J=1,M                                                    0325
      DO 26 K=1,M                                                    0326
   26 TQ(I,J)=TQ(I,J)+TP(I,K)*TT(K,J)                                0327
      DO 27 I=1,M                                                    0328
       DO 27 J=1,M                                                   0329
      TT(I,J)=0.                                                     0330
      DO 27 K=1,M                                                    0331
   27 TT(I,J)=TT(I,J)+TP(I,K)*TP(K,J)                                0332
      DO 10 I=1,M                                                    0333
      DO 10 J=1,M                                                    0334
   10 TP(I,J)=TT(I,J)                                                0335
      DO 21 I=1,M                                                    0336
      DO 21 J=1,M                                                    0337
   21 P(I,J)=TQ(I,J)                                                 0338
      PRINT 57                                                       0339
      DO 31 I=1,M                                                    0340
   31 PRINT 55,(P(I,J),J=1,M)                                        0341
      PRINT 58                                                       0342
      DO 32 I=1,M                                                    0343
   32 PRINT 55,(PHI(I,J),J=1,M)                                      0344
      DO 8 K=1,NTAU                                                  0345
```

```
      YK(K)=P(1,1)                                                          0346
      DO 9 I=1,M                                                            0347
      T(I)= 0.                                                              0348
      DO 9 J=1,M                                                            0349
    9 T(I)=T(I)+PHI(I,J)*P(1,J)                                             0350
      DO 8 I=1,M                                                            0351
    8 P(1,I)=T(I)                                                           0352
      A=YK(1)                                                               0353
      DO 3 K=1,NTAU                                                         0354
    3 YK(K)=YK(K)/A                                                         0355
      CALL DRAW(NTAU,XK,YK,MOD,0,LAB,ITITLE,0,0,0,0,0,0,5,7,1,LAST)         0356
      DO 4 I=1,NTAU                                                         0357
      R(I)=0.                                                               0358
      DO 4 J=1,3000,KDEL                                                    0359
    4 R(I)=R(I)+ZD(J)*ZD(J+3*I-3)                                           0360
      A=R(1)                                                                0361
      DO 5 I=1,NTAU                                                         0362
    5 R(I)=R(I)/A                                                           0363
      MOD=3                                                                 0364
      LAB=4H RS                                                             0365
      CALL DRAW(NTAU,XK,R ,MOD,0,LAB,ITITLE,0,0,0,0,0,0,5,7,1,LAST)         0366
      READ 56,(ITITLE(I),I=1,6)
      READ 56,(ITITLE(I),I=7,12)
      MZ=NTAU+1
      CALL PSD(YK,DEL,NTAU,0,FREQ,X)
      DO 44 J=1,MZ
   44 X(J)=X(J)*(-1.0)
C     THE SIGN INVERSION ON X(K) IS A CONVENIENCE FOR PLOTTING
      MOD=1
      LABEL=4H SDM
      CALL DRAW(MZ,X,FREQ,MOD,0,LABEL,ITITLE,0,0,0,6,0,2,6,8,1,LAST)
      CALL PSD( R,DEL,NTAU,0,FREQ,X)
      DO 45 J=1,MZ
   45 X(J)=X(J)*(-1.)
      MOD=3
      LABEL=4H SDS
```

```
      CALL DRAW(MZ,X,FREQ,MOD,0,LABEL,ITITLE,0,0,0,6,0,2,6,8,1,LAST)
      END                                                              0150
      SUBROUTINE PSD(A,DELTAT,M,IPRINT,FREQ,X)
      DIMENSION A(100),X(100),FREQ(100),TAU(100),ITITLE(12)
C     CALLING ARGUMENTS FOR S/R PSD
C         A = AUTO(CORRELATION WITH A(1) AT TAU = 0
C    DELTAT = TIME SPACING BETWEEN CORR. SAMPLES IN S E C O N D S .
C         M = NUMBER OF CORRELATION SHIFTS
C    ITITLE = TITLE OF PSD GRAPH IF DESIRED
C    IPRINT = FLAG SET TO 1 IF YOU WISH PRINT OUT PSD
C
C
  103 FORMAT(53H1     T U K E Y    S P E C T R U M    E S T I M A T E//)
  104 FORMAT(8H IDENT= O16,8H   M  = I4,9H DELTAT= F9.6 //)
  105 FORMAT(8H XFACT= F8.5,16H         A(0) = E12.5 //)
  106 FORMAT(53H   TAU(SEC)       R(TAU)      FREQ(CPS)     X(FREQ)    )
  107 FORMAT(1X,F11.7,3X,F10.5,3X,F9.3,3X,F10.5)
  108 FORMAT(8H  COMB= F9.5,15H    FMAX CPS = F12.3 //)
C
C
C     FIND  X(1)---THE POWER SPECTRAL DENSITY AT FREQ = 0.0 -----
   50 ASUM = 0.0 $ FM = M $ PI = 4.0*ATANF(1.0)
      CS1 = COSF(PI/FM) $ SN1 = SINF(PI/FM)
      CSL=CS1 $ SNL=SN1
C
      DO 52 L=2,M
      AZ=(1.0 + CSL) $ ASUM = ASUM + AZ*A(L)
      CSL1=CSL*CS1-SNL*SN1 $ SNL1=SNL*CS1+CSL*SN1
      CSL=CSL1 $ SNL=SNL1
   52 CONTINUE
C
      X(1) = 0.5*(ASUM + A(1))/FM
C     FIND X(K)---POWER SPECTRUM AT K=2,M
      CSK=CS1 $ SNK=SN1 $ MZ = M + 1
C
   53 DO 59 K=2,MZ
```

```
      ASUM=0.0 $ CSKL=CSK $ SNKL=SNK $ CSL=CS1 $ SNL=SN1
   54     DO 55 L=2,M
          AZ=(1.0+CSL)*CSKL $ ASUM=ASUM+AZ*A(L)
         CSL1=CSL*CS1-SNL*SN1 $ SNL1=SNL*CS1+CSL*SN1
          CSL=CSL1 $ SNL=SNL1
          CSKL1=CSKL*CSK-SNKL*SNK $ SNKL1=SNKL*CSK+CSKL*SNK
          CSKL=CSKL1 $ SNKL=SNKL1
   55     CONTINUE
      IF(K-MZ) 56,57,57
   56 DZ=1.0/FM $ GO TO 58
   57 DZ=0.5/FM $ GO TO 58
   58 X(K)=DZ*(ASUM+A(1))
      CSK1=CSK*CS1-SNK*SN1 $ SNK1=SNK*CS1+CSK*SN1
      CSK= CSK1 $ SNK=SNK1
   59 CONTINUE
C
C     X(K) IS THE POWER SPECTRAL DENSITY AT FREQ. 0.0 CPS TO FMAX CPS
C     APPLY TRAP. RULE TO FIND ENERGY CONTAINED IN POWER SPECTRUM
C     FOR RANGE OF K=1,M+1 - - - -  DEFINE ENERGY AS XENGY
C     NOTE-- X(K) IS ENERGY W.R.T. UNIT CHANGE OF INDEX K , NOT CPS ----
      ASUM=0.0
   65 DO 66 K=2,M
      ASUM=ASUM+X(K)
   66 CONTINUE
      XENGY=0.5*(X(1)+2.0*ASUM+X(M+1))
C     FIND FRACTION OF TOTAL ENERGY IN CALCULATED FREQ. RANGE
      XFACT=XENGY/A(1)
C     XFACT SHOULD = 1.0 IF ALL FREQUENCIES HAVE BEEN ACCOUNTED FOR...
C     OBTAIN SPECTRAL DENSITY WRT. CPS---IE., NORMALIZE WRT XENGY
C
C
      FREQ(1)=0.0 $ TAU(1)=0.0 $ MZ=M+1
      AZ=2.0*DELTAT*FM $ COMB=1.0/AZ
   67 DO 68 K=1,MZ
      X(K)=AZ*X(K)/XENGY $ FREQ(K+1)=FREQ(K)+COMB
      TAU(K+1)=TAU(K)+DELTAT
```

```
   68 CONTINUE
C
C      POWER SPECTRUM WRITE-OUT INSTRUCTIONS---
       PRINT 105,XFACT,A(1)
       PRINT 108, COMB, FREQ(MZ)
       JTEST=0
       IF(IPRINT-1) 40,41,40
   41  PRINT 106 $ PRINT 107 ,(TAU(K),A(K),FREQ(K),X(K),K=1,MZ) $ JTEST=1
   40  CONTINUE
       END
C
C  SUBROUTINE RTPLSUB (CO-OP ID - C2-NPGS-RTPLSUB  (F-60, F-63))
C  IS A STANDARD SUBROUTINE TO FIND REAL AND IMAG ROOTS OF A
C  POLYNOMIAL WITH REAL COEFFICIENTS USING BAIRSTOW AND NEWTON-
C  RAPHSON METHODS

C  SUBROUTINE DRAW  (CO-OP ID - J7-NPS-DRAW  (F-63)) IS A
C  GENERAL GRAPH OUTPUT SUBROUTINE.
```

APPENDIX V

(PROGRAM WRITTEN IN FORTRAN 63 FOR CDC 1604)

```
-COOP,,POPE J W BOX P, I/1/0/49/S/1S/2S/E/45=54,10,2000.
-FTN,L,E,R,N.
      PROGRAM TESTQ
C  THIS PROGRAM COMPUTES LIKELIHOOD FUNCTIONS OF 100 SAMPLES OF A
C  RANDOM TIME SERIES INPUT AS DATA, USING AUTOREGRESSIVE MODELS
C  OBTAINED BY PROGRAM MODEL.  A PRINCIPAL FUNCTION  OF THE PROGRAM
C  ISTO TEST ABILITY OF MODELS TO REPRESENT THE SIGNAL PLUS NOISE
C  ENERGY RECEIVED.  HENCE THE INPUT DATA BLOCK (4000 SAMPLES) IS
C  NORMALIZED TO HAVE A MEAN SQUARE VALUE OF UNITY, AND LIKELI-
C  HOOD FUNCTIONS  ARE EVALUATED FOR MODEL EXCITATIONS PROVIDING
C  VARIOUS PROPORTIONS OF THIS AMOUNT.  PROVISION IS MADE FOR
C  PROCESSING THE INPUT DATA WITH TWO DIFFERENT MODELS.  THE COMPUTED
C  LIKELIHOOD FUNCTION S FOR VARIOUS PROPORTIONS TRIED ARE GRAPHI-
C  CALLY PLOTTED AS A FUNCTION OF THE NUMBER OF SAMPLES PROCESSED.
      DIMENSION IBLOCK(1001),KDATA(4000,2),ZD(4000),PHI(12,12),
     1PHEEA(12),P10A(12,12),X10A(12),ALS(100),ALP(100),
     2PHEEB(12),P10B(12,12),X10B(12),BLS(100),BLP(100),
     3XK(100),ITITLE(12)
      COMMON/A/KDATA,IBLOCK
      COMMON/B/PHI,R,N
   50 FORMAT(4I10)
   51 FORMAT(2E10.2)
   52 FORMAT(8E10.2)
   53 FORMAT(O16)
   54 FORMAT(21H ERROR IN DATA INPUT )
   55 FORMAT(26H SAMPLE AVERAGE REMOVED = , E10.3)
   56 FORMAT(6A8)
   57 FORMAT(8H SAMPLES,3X,4H BLS,3X,4H BLP)
   58 FORMAT(I4,6E10.2)
   59 FORMAT(32H NR OF SAMPLES TO BE PROCESSED = ,I4)
   60 FORMAT(29HOLIKELIHOOD USING NOISE MODEL)
   61 FORMAT(8H RATIO = ,F6.2)
```

```
      READ 50,N,NK,KDEL,KSTART
C  N IS ORDER OF SIGNAL PROCESS MODELS
C  NK/KDEL IS NUMBER OF SAMPLES TO BE PROCESSED
C  KDEL EFFECTIVELY SETS SAMPLING RATE, AND MUST
C  AGREE WITH VALUE OF KDEL USED IN OBTAINING MODEL
C  KSTART IS NUMBER OF FIRST SAMPLE OF BLOCK TO BE PROCESSED
      NT=NK/KDEL
      READ 52,(PHEEA(I),I=1,N)
      READ 52,(PHEEB(I),I=1,N)
C  READ IN SIGNAL DATA AND NORMALIZE
      DO 100 KSIG=1,2
      READ 53,IDENT
      MAX=4000
      KLIST=1
      CALL DATA(IDENT,MAX,KLIST,KFLAG)
      IF(KFLAG)1,2,1
    1 PRINT 54
      GO TO 100
    2 CONTINUE
      ZAV=0.
      DO 3 K=1,MAX
      ZD(K)=KDATA(K)
    3 ZAV=ZAV+ZD(K)
      XMAX=MAX
      ZAV=ZAV/XMAX
      PRINT 55,ZAV
      RO=0.
      DO 4 K=1,MAX
      ZD(K)=ZD(K)-ZAV
      RO=RO+ZD(K)**2
    4 CONTINUE
      RO=RO/XMAX
      SIGMA=SQRTF(RO)
      DO 10 K=1,MAX
   10 ZD(K)=(ZD(K)/SIGMA)/1.414
C  GENERATE BASIC PHI MATRIX
```

```
      DO 5 I=1,N
      DO 5 J=1,N
    5 PHI(I,J)=0.
      DO 6 J=2,N
      I=J-1
    6 PHI(I,J)=1.

      READ 56,(ITITLE(I),I=1,6)
      READ56,(ITITLE(I),I=7,12)
      PRINT 59, NT
      KQ=1
   25 CONTINUE
      GO TO (11,12,13,14,15,16,17,18,19,200)KQ
   11 QA=.0419      $LAB=4H.1   $R=.9
      MOD=I
      GO TO 20
   12 QA=.0838      $LAB=4H.2   $R=.8
      MOD=2
      GO TO 20
   13 QA=.1252      $LAB=4H.3   $R=.7
      GO TO 20
   14 QA=.1672      $LAB=4H.4   $R=.6
      GO TO 20
   15 QA=.2090      $LAB=4H.5   $R=.5
      GO TO 20
   16 QA=.2510      $OAB=4H.6   $R=.4
      GO TO 20
   17 QA=.2925      $LAB=4H.7   $R=.3
      GO TO 20
   18 QA=.3340      $LAB=4H.8   $R=.2
      GO TO 20
   19 QA=.3760      $LAB=4H.9   $R=.1
       MOD=3
   20 CONTINUE
      CALL STEADY (PHEEA,QA,P10A)
      DO 21 I=1,N
```

```
 21 X10A(I)=0.
    ZT1LA=ZD(KSTART)-X10A(1)    $ A=P10A(1,1)+R
    ALS(1)=ZT1LA**2/A
    ALP(1)=LOGF(A)
    DO 22 K=2,NT
    KD=KSTART+3*K-3
    CALL LCOMP(PHEEA,QA,P10A,X10A,ZT1LA,ZD(KD),ALS(K),ALP(K)
   1,ALS(K-1),ALP(K-1))
 22 CONTINUE
    RATIO=1.-R
    PRINT 60
    PRINT 61, RATIO
    PRINT 57
    PRINT 58,NT,ALS(NT),ALP(NT)
    DO 23 I=1,NT  $ALS(I)=ALS(I)+ALP(I)  $XK(I)=I
 23 CONTINUE
    CALL DRAW(NT,XK,ALS,MOD,0,LAB,ITITLE,0,0,0,0,0,0,5,7,1,LAST)
    KQ=KQ+1
    GO TO 25
200 CONTINUE

    READ 56,(ITITLE(I),I=1,6)
    READ56,(ITITLE(I),I=7,12)
    PRINT 59, NT
    KQ=1
 26 CONTINUE
    GO TO(31,32,33,34,35,36,37,38,39,100)KQ
 31 QB=.0474      $  LAB=4H .1     $ R=.9
    MOD=1
    GO TO 40
 32 QB=.0946      $  LAB=4H .2     $  R=.8
    MOD=2
    GO TO 40
 33 QB=.142       $  LAB=4H .3     $  R=.7
    GO TO 40
 34 QB=.1895      $  LAB=4H .4     $  R=.6
```

```
      GO TO 40
   35 QB=.237       $LAB=4H.5   $R=.5
      GO TO 40
   36 QB=.284       $LAB=4H.6     $R=.4
      GO TO 40
   37 QB=.3315      $LAB=4H.7   $R=.3
      GO TO 40
   38 QB=.379       $LAB=4H.8   $R=.2
      GO TO 40
   39 QB=.426       $LAB=4H.9   $R=.1
       MOD=3
   40 CONTINUE
      CALL STEADY (PHEEB,QB,P10B)
      DO 7 I=1,N
    7 X10B(I)=0.
      ZT1LB=ZD(KSTART)-X10B(1)  $ A=P10B(1,1)+R
      BLS(1)=ZT1LB**2/A
      BLP(1)=LOGF(A)
       DO 8 K=2,NT
      KD=KSTART+3*K-3
      CALL LCOMP(PHEEB,QB,P10B,X10B,ZT1LB,ZD(KD),BLS(K),BLP(K)
     1,BLS(K-1),BLP(K-1))
    8 CONTINUE
      RATIO=1.-R
      PRINT 60
      PRINT 61, RATIO
      PRINT 57
      PRINT 58,NT,BLS(NT),BLP(NT)
      DO 9 I=1,NT           $ ALS(I)=ALS(I)+ALP(I)    $ XK(I)=I
      BLS(I)=BLS(I)+BLP(I)
    9 CONTINUE
      CALL DRAW(NT,XK,BLS,MOD,0,LAB,ITITLE,0,0,0,0,0,0,5,7,1,LAST)
      KQ=KQ+1
      GO TO 26
  100 CONTINUE
      END
```

```
      SUBROUTINE STEADY (PHEE,Q,P10)
      DIMENSION PHI(12,12),
     1PHEE(12),P10(12,12),TQ(12,12),TP(12,12),TT(12,12)
      COMMON/B/PHI,R,N
   50 FORMAT(1X,8E10.2)
   51 FORMAT(20X,4H PHI )
   52 FORMAT(10X,31H STEADY-STATE COV MATRIX OF SGP )
      DO 1 I=1,N  $  DO 1 J=1,N
      TQ(I,J)=0.
    1 TP(I,J)=PHI(I,J)
      TQ(N,N)=Q
      DO 2 I=1,N
    2 TP(N,I)=PHEE(I)
      PRINT 51
      DO 3 I=1,N
    3 PRINT 50,(TP(I,J),J=1,N)
      DO 10 KK=1,10
      DO 4 I=1,N  $ DO 4 J=1,N
      TT(I,J)=0.
      DO 4 K=1,N
    4 TT(I,J)=TT(I,J)+TQ(I,K)*TP(J,K)
      DO 5 I=1,N  $  DO 5 J=1,N  $ DO 5 K=1,N
    5 TQ(I,J)=TQ(I,J)+TP(I,K)*TT(K,J)
      DO 6 I=1,N  $ DO 6 J=1,N
      TT(I,J)=0.  $ DO 6 K=1,N
    6 TT(I,J)=TT(I,J)+TP(I,K)*TP(K,J)
      DO 10 I=1,N  $ DO 10 J=1,N
   10 TP(I,J)=TT(I,J)
      DO 7 I=1,N  $ DO 7 J=1,N
    7 P10(I,J)=TQ(I,J)
      PRINT 52
      DO 8 I=1,N
    8 PRINT 50,(P10(I,J),J=1,N)
      END

      SUBROUTINE LCOMP(PHEE,Q,P10,X10,ZT1L ,ZK,XLS,XLP,XLS1,XLP1)
```

```
      DIMENSION PHI(12,12),PHEE(12),
     1P10(12,12),X10(12),P(12,12),G(12),XST(12),TT(12,12)
      COMMON/B/PHI,R,N
      DO 1 I=1,N
      PHI(N,I)=PHEE(I)
    1 G(I)=P10(I,1)/(P10(1,1)+R)
      DO 2 I=1,N $ DO 2 J=1,N
    2 P(I,J)=P10(I,J)-G(I)*P10(1,J)
      DO 3 I=1,N
    3 XST(I)=X10(I)+G(I)*ZT1L
      DO 4 I=1,N
      X10(I)=0.
      DO 4 J=1,N
      X10(I)=X10(I)+PHI(I,J)*XST(J)
      TT(I,J)=0.
      DO 4 L=1,N
    4 TT(I,J)=TT(I,J)+PHI(I,L)*P(L,J)
      DO 5 I=1,N  $ DO 5 J=1,N
      P10(I,J)=0. $ DO 5 L=1,N
    5 P10(I,J)=P10(I,J)+TT(I,L)*PHI(J,L)
      P10(N,N)=P10(N,N)+Q
      ZT1L=ZK-X10(1)     $ A=P10(1,1)+R
      XLP=XLP+LOGF(A)
      XLP=    LOGF(A)        +XLP1
      XLS=XLS+(ZT1L**2)/A
      XLS=    (ZT1L**2)/A       +XLS1
      END
```

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1. ORIGINATING ACTIVITY (Corporate author)
U. S. Naval Postgraduate School
Monterey, California 93940

2a. REPORT SECURITY CLASSIFICATION
Unclassified

2b. GROUP

3. REPORT TITLE

Real-Time Estimation and Random Signal Detection

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)
Doctor of Philosophy Thesis in Electrical Engineering - May 1966

5. AUTHOR(S) (Last name, first name, initial)

Pope, Jr., John W. R. Lieutenant Commander U. S. Navy

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
May 1966	136	40

8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)
b. PROJECT NO.	
c.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
d.	

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This work investigates some problems arising in application of (Kalman) linear filter theory to real problems, where practical estimates must replace exact theoretical quantities in problem formulation. The principle objective is application of linear filter theory to random signal detection/classification. Howdver, an example of classical estimation, error estimation in shipboard inertial navigation systems, is offered to illustrate general points discussed. A unified treatment of models for random time series is presented, including a comparative review of models which have been proposed and procedures for obtaining model coefficients. Correlation detection of deterministic signals is discussed and the resulting principles extended to the case of random signal detection. Application of linear filter theory to the problem is indicated. Finally, an experimental study in random signal detection/classification is included. Experimental signals used are hydrophone recordings of sea noise and sea noise plus diesel submarine. Consistency of successful results obtained suggests practical utility of method in certain random signal detection/classification problems.

DD FORM 1 JAN 64 1473

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